A Hierarchical Production Planning and Finite Scheduling Framework for Part Families in the Flexible Job-shop (with a case study)

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Abstract

Tendency towards optimization in last decades has resulted in creating multi-product manufacturing systems. Production planning in such systems is difficult, because the calculated optimal production volume must be consistent with the limitation of production system. Hence, integration has been proposed to decide about these problems concurrently. Main problem in integration is how we can relate production planning in the medium-term timeframe to scheduling in the short-term timeframe. Our contribution creates production planning and scheduling framework in the flexible job-shop environment with respect to the time-limit of each machine in order to produce different part families in the automotive industry. Production planning and scheduling have an iterative relationship. In this strategy, information flow is transformed in a reciprocative way between production planning and scheduling in order to satisfy the time-limit of each machine. The proposed production planning has a heuristic approach and renders a procedure to determine the production priority of different part families based on the safety stock. Scheduling is performed with ant colony optimization and assigns machines in order of priority to different part families on each frozen horizon. Results showed that the proposed heuristic algorithm for planning decreased parts inventory at the end of planning horizon. Moreover, the results of the proposed ant colony optimization were near the optimal solution. The framework was performed to produce sixty-four different part families in the flexible job-shop with fourteen different machines. The output of the approach determined the volume of production batches for part families on each frozen horizon and assigned different operations to machines.

Keywords: Production Planning, Finite Scheduling, Part Families, Flexible Job-shop.

1. Introduction

Current production and inventory control systems consist of make-to-order (MTO), make-to-stock (MTS) and MTS-MTO systems. The MTS-MTO system is used when there are various products. In such systems parts-making is performed by the MTS system while the assembly of products is done by the MTO system (see Figure 1). Production planning and scheduling frameworks in such systems are of two types of hierarchical and integrated. In the first approach, planning and scheduling are performed hierarchically and in the second approach, planning and scheduling are done simultaneously. In this paper, we propose a hierarchical approach for production planning and finite scheduling in MTS-MTO systems.

Our proposed production planning has a heuristic approach and is performed based on required lots of part families on the planning horizon. Parameters such as lot size, safety stock and safety lead-time are considered. Lot size is calculated based on the periodic order quantity (POQ)

The scheduling of different production batches in the Flexible Job-shop (FJS) environment is studied with regard to the Independent setup time.

Scheduling is performed with the Ant Colony Optimization (ACO) based on required lots of part families on the frozen horizon.

This paper studies hierarchical production planning and scheduling, which have an iterative relationship, for the components of different products in the automotive industry.

Our contribution creates production planning and scheduling framework in the FJS environment with the consideration of time-limit in order to produce different part families. This framework is evaluated in Safe Sanat, a supplier of Iran Khodro and SAIPA Company.

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The paper is organized as follows. In the following section we review previous related work on production planning and scheduling. Section (3) contains an extended integer programming formulation of the FJS problem with considering the time-limit of each machine. Then Section (4) proposes a hierarchical production planning and finite scheduling in the FJS environment.

A case study in Safe Sanat Company is explained in Section (5), and in Section (6), we report the results of the proposed hierarchical production planning and scheduling. Finally, we conclude the paper with a summary and some directions for future research in Section (7).

2. Literature Review

2.1. Uncertainty in Production and Inventory Control Systems

Production in MTS systems without considering issues such as demand uncertainty for finished product, material procurement, fluctuation in production time, machine breakdown, poor quality of products and failure to provide material in a timely manner is not possible. Controllable parameters in uncertainty conditions are safety stock, safety lead-time, production batch, frozen horizon and planning horizon.

2.2. Important Parameters in Production Planning

2.2.1. Safety stock, Safety Lead-time and Setup time

Safety stock parameter decreases the probability of shortage, but increases holding costs. It is calculated by minimizing the sum of holding and shortage costs based on the significance level. Melnyk and Piper (1981) state that safety lead-time is equivalent to safety stock in which time is used instead of quantity. It is usually considered as a standard variance of $k$ lead-times. According to Whybark and Williams (1976), safety stock is used when the uncertainty is based on quantity and safety lead-time is used when the problem is faced with the estimation of lead-time.

In this study we consider production planning and scheduling in MTS systems. For such systems, Molinder (1997) considers setup time, significance level, inventory and shortage cost. He proposes safety stock for the high variation of demand and the low variation of production lead-time. He also proposes safety lead-time for the high variation of demand and lead-time.

Internal safety lead-time is considered for machine breakdown, poor quality of products and variation in production time while external safety lead-time is considered for the activity done outside the organization. In this article we use external safety lead-time to calculate the safety stock. They consist of galvanized plating, smarth, electroplates dakrvmat and heat treatment. These operations start when internal activities end.

2.2.2. Lot-size

First, the main method of determining lot size is economic order quantity (EOQ). This method calculates the fixed order size through Wilsons' formula. But the interval between two consecutive orders may be variable. The second method is POQ in which an optimized fixed interval between two orders is calculated and using the interval, the order size of each period is calculated. The third method is Wagner Whithin algorithm. It is based on minimizing order costs for dynamic demands, without considering capacity constraints. Due to the long duration of solution, Silver Meal, Least Unit Cost (LUC) and Part Period balancing (PPB) techniques are used instead of Wagner Whithin.

In this regard, we propose a heuristic procedure to calculate production batches based on the POQ method.

2.2.3. Planning Horizon and Frozen Horizon

To maintain a balance between a suitable significance level for customer satisfaction and decreasing inventory, lots of techniques use frozen horizon for production planning. Using frozen horizon ($FH$) decreases the significance level and inventory. In this article, we divide the planning horizon ($PH$) into several frozen horizons.

2.3. Production Scheduling Systems in Job-shop and Flexible Job-shop

A classical job-shop (JS) system consists of a set of different machines that is used for operation on all the jobs. Each job specifies the processing order through the machines, i.e., a job is composed of an ordered list of operations. Each operation is determined by the machine required and the processing time of it. There is no job-preemption and each machine can handle only one...
operation at a time. Each operation can also be performed only on one machine. Moreover, in this JS, the sequence of operations for each job is fixed and the problem is to find the job sequences on the machines which minimize the Make-span, i.e. minimizing the maximum of the completion time for all operations. It should be pointed out that Garey et al. (1976) proved that job-shop problem is a NP-hard.

FJS is another approach in which each operation can be performed on more than one machine. In this strategy, all parts from different families are processed in one job center (cell) and there is no more than one shop center. It is used for non-stop production processes. In this approach, the machines with a similar usage can handle a specified operation. Because of its multi-route for assigning each operation to a machine, FJS is more complex than JS and is strongly NP-hard. FJS' heuristics are either hierarchical or integrated. In the hierarchical approach, assigning and sequencing operations on each machine are performed separately; in other words, assignment and sequence of operations are considered independent of each other. In the integrated approach, assigning and sequencing operations are performed simultaneously and they are interdependent. There are some studies on FJS systems. The earliest finishing time (EFT) rule with respect to alternative operation to minimize mean flow time in such a system was investigated by Nasr and Elsayed (1990).

Mahmood et al. (1990) developed dynamic scheduling heuristics to stress good due date performance while reducing overall setup time in a job-shop cell. Tsai and Li (2000) presented a due date-oriented scheduling heuristic algorithm for job-shop cell manufacturing systems based on capacity constraint resource. Li and Wang (2012) suggested a pheromone-based approach using a multi-agent cell manufacturing system in which parts families can move between flexible routes in different job centers. Fattahi et al. (2007) presented three heuristic approaches for FJS scheduling problems for parts-making industries. Ponnambalam et al. (2010), the closest research to our study, developed an ant colony optimization approach in FJS with the consideration of relative pheromone trail between different operations on a specified machine. In this paper, we develop this research for different production batches.

Finite scheduling is about assigning no more operation to a machine which is expected to execute in a given time period. There is no research done about scheduling of different parts families in flexible job-shop with the consideration of capacity constraint of machines. We, therefore, propose a heuristic for the finite scheduling of different part families in the FJS systems.

2-3-1. Setup Time in Job-shop Scheduling Systems

To use setup times, there are two approaches. In the first one, the setup time for each operation is independent of the previous operation on an identified machine. In the second approach, the setup time for each operation is dependent on the previous operation on an identified machine.

2.4. A Mixed integer linear formulation for Flexible Job-shop Scheduling

In this study, we develop an MILP model with consideration of time-limit for production batches on each frozen horizon based on Mehrabad and Fattahi (2007).

2.5. Production Planning and Scheduling Framework

The most important research about production planning and scheduling framework was done by Meybodi (1994), in which the integration of production activity control with consideration of final customer demand was studied. A heuristic algorithm with respect to family’s production cycle time was proposed. Furthermore, production orders for families were presented, and the lot size was calculated using the EOQ model.

2.6. The Study

This paper studies production planning and scheduling for the components of different products in the automotive industry in MTS-MTO systems. There is no identified production planning and scheduling framework for part families considering time-limit. Hence, to produce different parts families, we look for creating a production planning and scheduling framework with respect to the time-limit of each machine. To do so, we go through three stages. First, we propose a heuristic to calculate the production batches of parts families in each frozen horizon. Second, we develop the ACO method in the FJS system and then compare it with the developed mixed integer linear programming. Third, we propose a hierarchical production planning and finite scheduling framework for parts families in the FJS system. This strategy is used in Safe Sanat to calculate its effectiveness.

3. The Mixed Integer Linear Formulation for Flexible Job-shop Scheduling

The production batch consists of K lots related to a certain family. Each lot pertains to an identified part. All lots are stacked to reach an equal size. Each time all parts are processed by a certain machine, and then they are added to their lots in order to reach the specified quantity. On the other hand, we have the batch availability with K lot sizes. Finally, after the production batch is prepared for next operation, it is transferred to a new machine. The assumptions of this model are as follows:

1. The production batch and transferring batch for each family are equal in a certain frozen horizon.
2- Transportation times between different machines are not considered.
3- The setup time for a new production batch on a certain machine is independent of the previous production batch on the same machine.
4- There is no job-preemption.
5- The number of different machines that can be used for each operation for an identified production batch is between 1 to 3.
6- The setup for all lots of a production batch is performed concurrently.
7- The preliminary setup for production batches is not considered.
8- The possibility of batch splitting for simultaneous production on different machines is not considered.

Here are the notations for scheduling of the production batches:

\[ \begin{align*}
\text{n} & \quad \text{the number of machines} \\
\text{m} & \quad \text{the number of production batches} \\
\text{h}_{j'} & \quad \text{operation number} \\
\text{i}' & \quad \text{index of machines, i}' = 1, \ldots, n \\
\text{j}', \text{k} & \quad \text{index of production batch, j}' = 1, \ldots, m \\
\text{h}, \text{l} & \quad \text{index of operation, h = 1, \ldots, h}_{j'} \\
\text{p}_{j',j,h} & \quad \text{process time for operation h of production batch j}' on machine i'
\end{align*} \]

\[ \begin{align*}
\text{s}_{j',j,h,k,l} & \quad \text{setup time for operation l of production batch k on machine i'} \\
\text{RT}_{i'} & \quad \text{regular time for machine i'} \\
\text{ET}_{i'} & \quad \text{extra time for machine i'} \\
\text{M} & \quad \text{a large number}
\end{align*} \]

And the integer programming uses the following variables:

\[ \begin{align*}
\text{c}_{\text{max}} & \quad \text{maximum completion time for production batch} \\
\text{t}_{j',h} & \quad \text{start time of operation h of production batch j}' \\
\text{f}_{j',h} & \quad \text{finish time of operation h of j}' \text{ equal to 1 if operation h of production batch j}' is assigned to machine i', 0 otherwise; \\
\text{y}_{j',j',h,k,l} & \quad \text{equal to 1 if operation l of production batch k after operation h of production batch j}' is assigned to machine i', 0 otherwise; \\
\text{a}_{j',j',h} & \quad \text{equal to 1 if operation h of production batch j}' can be performed on machine i', 0 otherwise; \\
\end{align*} \]

Using the above parameters and variables, we can represent our problem of minimizing Make-span to a MILP as follows:

\[ \begin{align*}
\text{min } & \quad \text{C}_{\text{max}} \\
\text{subject to } & \quad \text{f}_{j',h} \leq \text{c}_{\text{max}} \quad \forall j' = 1, \ldots, m; h = 1, \ldots, h_{j'} \\
\text{f}_{j',h} & \leq \text{t}_{j',h} - 1 \\
\text{t}_{j',h} & \geq 0 \quad \forall j' = 0, \ldots, m; i' = 1, \ldots, n; h = 1, \ldots, h_{j'} \\
\text{f}_{j',h} & \leq \text{t}_{j',h} + \text{RT}_{i'} + \text{ET}_{i'} \\
\text{y}_{j',j',h,k,l} & \geq 0 \quad \forall j', h = 1, \ldots, h_{j'} \\
\text{y}_{j',j',h,k,l} & \leq 0 \quad \forall j', h = 1, \ldots, h_{j'} \\
\text{a}_{j',j',h} & \leq \text{y}_{j',j',h,k,l} \quad \forall j', h = 1, \ldots, h_{j'} \\
\text{x}_{j',j',h,k,l} & \leq \text{y}_{j',j',h,k,l} \quad \forall j', h = 1, \ldots, h_{j'} \\
\text{x}_{j',j',h,k,l} & \leq 0 \quad \forall j', h = 1, \ldots, h_{j'} \\
\end{align*} \]
\[ \forall l = 1, \ldots, h_k; \forall l' = 1, \ldots, n \]

Eq. (1) means that this problem is to minimize the Make-span. Equations (2), (3) guarantee that each production batch has an identified sequence of operations. Eq. (4) defines the Make-span, and Eq. (5) assures that the operation \( h \) of production batch \( j' \) can process on alternative machines. Equations (6), (7) assure that in any time one operation can process on each machine. Equations (8) and (9) show the possibility of sequencing operations for different families on each machine. Eq. (10) says that that the operation \( h \) of production batch \( j' \) is performed on only one machine. Equations (11) and (12) guarantee that only one operation is performed on a certain machine when the operation which is processing on it is fully performed. Finally, Eq. (13) shows the time-limit for each machine on a frozen horizon.

4. The Proposed Approach for Hierarchical Production Planning and Finite Scheduling

4.1. The Production Planning Approach for Different parts families

First, parts were divided into different families with respect to the bill of material. All parts in any family had similar processes. Production planning for families were performed based on product demand forecast and usage rate of its parts. Then the actual start of production for each family was calculated based on the run-out date of parts inventory. Finally, the production batch for each family in any frozen horizon was obtained. Internal and external safety lead-time was also considered to calculate the safety stock for each part. The assumptions of the production planning system were as follows:

1- Production planning was performed based on the MTS system.
2- Production planning was performed on the planning horizon.
3- The lot size for families was calculated based on the POQ approach.
4- There was uncertainty for finished-products demand and external lead-time.
5- Shortage was not considered on the planning horizon.

4.1.1. Steps of the Heuristic Production Planning Approach

Step (1) – Forecasting the parts demand of different products is calculated by (19).

\[ i_k = \sum_{i=1}^{n} U_{ijk} D_i \cdot \delta_{ijk} \]  \hspace{1cm} (19)

where

\[ U_{ijk} \] usage rate for the part \( k \) of family \( j \) in product \( i \)

\[ D_i \] forecasted demand of product \( i \) at the beginning of planning horizon

\[ \delta_{ijk} \] equal to 1 if the part \( k \) of family \( j \) belongs to product \( i \), 0 otherwise

Step (2) – Calculating the safety stock for the part \( k \) of family \( j \) based on the fixed order interval system by (20), (21) and (22).

\[ SS_{jk} = \text{ROUNDUP}\left[ Z_a \times (\sigma_{D_{L+T}})_{jk} \right] \]  \hspace{1cm} (20)

where

\[ SS_{jk} \] safety stock for the part \( k \) of family \( j \) during lead-time plus planning period

\[ (\sigma_{D_{L+T}})_{jk} \] standard deviation for the part \( k \) of family \( j \) during lead-time plus planning period

\[ L'_{jk} \] average production lead-time for the part \( k \) of family \( j \)

\[ L_{jk} \] average external safety lead-time for the part \( k \) of family \( j \)

\[ T \] planning horizon period(month)

\[ PHSD \] planning horizon start date

\[ PHFD \] planning horizon finished date

\[ \sigma^2_{D_{L+T}} \] forecasted demand variance for the part \( k \) of family \( j \)

\[ \sigma^2_{(L' + T + L')_{jk}} \] lead-time variance for the part \( k \) of family \( j \) during planning horizon

\[ FD_{jk} \] forecasted demand for the part \( k \) of family \( j \) during planning horizon
\( Z_\alpha \) confidence level \( \alpha \)

Step (3) – Calculating the run-out date of inventory for each part at the beginning of planning horizon by (23):

\[
F_{jk} = \frac{FD_{jk}}{W_D} \tag{23}
\]

\( ; \forall j = 1, ..., m; \forall k = 1, ..., l_j \)

where

- \( I_{jk} \) on-hand inventory for the part \( k \) of family \( j \) at the beginning of planning horizon
- \( F_{jk} \) daily usage rate for the part \( k \) of family \( j \) at the beginning of planning horizon
- \( PHSD \) planning horizon start date
- \( W_p \) The number of days on the planning horizon

Step (4) – Calculating the actual lot size of each family by (24) and (25):

\[
ALS_j = \max_{k \in 1, 2, ..., l_j}(TLS_{jk}) \quad ; \forall j = 1, ..., m \tag{24}
\]

\[
TLS_{jk} = FD_{jk} - (I_{jk} - SS_{jk}) \quad ; \forall j = 1, ..., m; \forall k = 1, ..., l_j \tag{25}
\]

where

- \( TLS_{jk} \) temporary lot size for the part \( k \) of family \( j \)

Step (5) – Calculating the actual start of production for family \( j \) by (26):

\[
ASOP_j = \min_{k \in 1, 2, ..., l_j}(FROD_{jk}) + 1 \tag{26}
\]

where

- \( ASOP_j \) actual start of production date

Step (6) – Calculating the production batch per different frozen horizon by (27):

\[
SD_{FH} \leq ASOP_j \leq FD_{FH} \quad ; \text{if} \quad \exists Q \epsilon 1, ..., n \tag{27}
\]

\[
P_{BFH,j} = \text{ROUNDUP} \left( \frac{ALS_j}{W_D} \right) \quad ; \text{if} \quad FD_{FH} = ASOP_j \quad ; \forall j = 1, ..., m; \; P = Q
\]

\[
P_{BFH,j} = \text{ROUNDUP} \left( \frac{ALS_j}{W_D} \right) \cdot (FD_{FH} - ASOP_j + 1) \quad ; \text{if} \quad SD_{FH} < ASOP_j < FD_{FH} \quad ; \forall j = 1, ..., m; \; P = Q
\]

\[
P_{BFH,j} = \text{ROUNDUP} \left( \frac{ALS_j}{W_D} \right) \cdot (FD_{FH} - SD_{FH} + 1) \quad ; \forall j = 1, ..., m; \; \forall P = Q + 1, ..., n
\]

If the production batches for each family on different frozen horizons are considered equal, then we have (28):

\[
SD_{FH} \leq ASOP_j \leq FD_{FH} \quad ; \exists Q \epsilon 1, ..., n
\]

\[
P_{BFH,j} = \text{ROUNDUP} \left( \frac{ALS_j}{n - Q + 1} \right)
\]

\( ; \forall j = 1, ..., m; \forall P = Q, Q + 1, ..., n \tag{28} \)

where

- \( P_{BFH,j} \) the production batch of family \( j \) on frozen horizon \( p \)
- \( PHFD \) planning horizon finished date
- \( SD_{FH} \) start date for frozen horizon \( Q \)
- \( FD_{FH} \) finished date for frozen horizon \( Q \)

4.2. The scheduling approach for different production batches with the developed Max- Min Ant system

Scheduling of different production batches was performed with a Max-Min ant system by considering the priority of assigning machines to each operation. The scheduling also had a hierarchical approach.

Notations of the ant colony optimization are as follows:

- \( n \) the number of production batch per frozen horizon
- \( i', i'' \) index of production batch \( i', i'' = 1, ..., m \)
- \( j', j'' \) index of operation \( j', j'' = 1, ..., j_i \)
- \( O_{i', j'} \) operation \( j \) of production batch \( i' \)
- \( z \) index of ant, \( z = 1, ..., u \)

4.2.1. Steps of the scheduling algorithm

For each ant in hierarchical approach, the steps are described as follows:

Step (1) – Obtaining the initial solution for assigning \( O_{i', j'} \) to machine \( r \) by (29) an (30).

\[
p_{i', j'}^{z} (m) = n \left[ \frac{a_{i', j'}^r \sum_{i'^{'} = 1}^{m} \left( a_{i'^{'}, j'}^r \right)^{\alpha} \left( \nu_{i'^{'} r}^z \right)^{\beta}}{\sum_{i'^{'} = 1}^{m} \sum_{r = 1}^{N} \left( a_{i'^{'}, j'}^r \right)^{\alpha} \left( \nu_{i'^{'} r}^z \right)^{\beta}} \right]_+(m)
\]
where

\[
p_{i,j,r}(tn) = \text{probability of allocation } O_{i,j} \text{ to route } r \]
\[
r = \text{index of route } r, r = 1, ..., r_{i',j'} \]
\[
K_{i',j'} = \text{machine number of } O_{i,j} \text{ for route } r \]
\[
\alpha_{i,j,r} = \text{priority ratio of assigning machine } K \text{ to } O_{i,j} \text{ in route } r \]
\[
\beta = \text{heuristic the parameters for controlling the relative importance of the pheromone trail and the heuristic information} \]
\[
\tau_{i,j,r}(tn) = \text{pheromone trail of } O_{i,j} \text{ on route } r \text{ in iteration } tn \]
\[
R_{i,j,r} = \text{the number of alternative machine for } O_{i,j} \]
\[
n_{i,j,r} = \frac{1}{\tau_{i,j,r} + S_{i,j,r}} \tag{30} \]

where

\[
n_{i,j,r} = \text{heuristic information from } T_{i,j,r} \]
\[
S_{i,j,r} = \text{setup time of route } r \text{ for } O_{i,j} \]
\[
T_{i,j,r} = \text{process time of route } r \text{ for } O_{i,j} \]

Step (2) - Allocating \( O_{i,j} \) to the machine

In the current iteration, if the selected \( O_{i,j} \) based on the earliest finishing time is unique on a machine, the ant will assign \( O_{i,j} \) to the machine. Then the assigned operation to a special machine will save in \( Q(i^2)(s) \) at each step. If there is conflict between at least two operations in one step, so that one of the operations has the earliest finishing time on the same machine, a operation will be assigned to the machine based on the following problem rule by (31).

\[
P_{i,j,r}''(tn) = \frac{[\xi_{k,i,j,r}(tn)]^\gamma \psi_i(s)]^\omega}{\sum_{i',j'} [\xi_{k,i',j',r}(tn)]^\gamma \psi_i(s)]^\omega} \forall (i',j') : O_{i,j} \in G_k(s) \tag{31} \]

where

\[
P_{i,j,r}''(tn) = \text{probability of allocation } O_{i,j} \text{ on machine } k \text{ after } O_{i,j} \]
\[
\xi_{k,i,j,r}(tn) = \text{pheromone trail of } O_{i,j} \text{ relative to } O_{i,j} \text{ on machine } k \text{ in iteration } tn \]
\[
\psi_i(s) = \text{sum of the processing time of all unassigned operation of production batch } j' \text{ at step } s \]
\[
\omega, \gamma = \text{heuristic parameters for controlling the pheromone trial} \]
\[
G_k(s) = \text{collection of operations with conflict on machine } k' \]

If there is conflict between some operations on a machine that has an operation with the earliest finishing time, firstly operations will be sorted randomly, and then the pheromone trail for each operation will be considered relative to the previous operation. Finally, after calculating the probability of allocation through Eq. (31), the operation will be selected based on generating a random number between 0 to 1 (see Figure 2). For each ant with these steps, a feasible scheduling is obtained that is considered as a Make-span.

Step (3) – The datum time for each operation to perform on machine \( k \) is obtained in each step as follows by (32):

\[
\text{Datum Time } (O_{i,j}k') = (T_{i,j,k} + S_{i,j,k}) + \text{Max}(DO_{i,j,k' \rightarrow}, DM_{i,j,k'}) \tag{32} \]

where

\[
DO_{i,j,k' \rightarrow} = \text{datum time for } O_{i,j} \text{ to the latest operation from a production batch that is assigned to machine } k, \text{ before assigning } O_{i,j} \text{ to this machine} \]

Step (4) - Sorting the answers:

The best solution in the current iteration (ibest) for all ants and the best solution from the beginning of the iteration (gbest) are sorted separately.

Step (5) – Terminating the check module:

A specified number of iterations with respect to the problem size is considered. When the number of iterations is reached to the specified number, the scheduling is terminated. Otherwise, the pheromone trials are updated and the scheduling procedure for production batches is repeated.

Step (6) – Updating the pheromone trials:

Step (6) /stage (1) – Updating the pheromone trail for operations:

For ibest at the end of each iteration, if \( O_{i,j} \) is assigned to route \( r \), pheromone updating is performed by Eq. (33) and (35). Otherwise, it is performed by Eq. (34) and (35).
\[ \forall i, i' = 1, ..., m; \forall j, j' = 1, ..., j'; \forall r = 1, ..., r_{ij} \]

\[ \tau_{i'j'}(tn + 1) = \rho \cdot \tau_{i'j'}(tn) \]

\[ \forall i, i' = 1, ..., m; \forall j, j' = 1, ..., j'; \forall r = 1, ..., r_{ij} \]

\[ \Delta \tau_{i'j'}(best) = \frac{1}{f(s_{best})} \]

where

\[ \rho \] evaporation factor between 0,1

\[ f(s_{best}) \] ibest or gbest

The pheromone trial range for each operation is obtained through Equations (36), (37), (38), (39), (40), (41) and (42) as follows:

\[ \tau_{i'j'}(tn) = \tau_{\max}(tn) \quad \text{if} \quad \tau_{i'j'}(tn) \geq \tau_{\max}(tn) \]

\[ \tau_{i'j'}(tn) = \tau_{\min}(tn) \quad \text{if} \quad \tau_{i'j'}(tn) < \tau_{\min}(tn) \]

\[ \tau_{i'j'}(tn) = \tau_{i'j'}(tn) \quad \text{if} \quad \tau_{\min}(tn) \leq \tau_{i'j'}(tn) \leq \tau_{\max}(tn) \]

\[ \tau_{i'j'}(1) = \tau_{\max}(1) \]

\[ \tau_{\max}(tn + 1) = \frac{1}{(1 - \rho) f(g_{\text{best}})} \]

\[ \tau_{\min}(tn + 1) = \frac{\tau_{\max}(tn + 1)}{y} \]

where

\[ \tau_{\max}(tn) \] maximum pheromone trial for each route in iteration tn

\[ \tau_{\min}(tn) \] minimum pheromone trial for each route in iteration tn

Step (6) /stage (2) – Updating the relative pheromone trail between operations

In the {ibest} at the end of iterations, if \( O_{i'j'} \), \( O_{i'j'} \) are processed sequentially on machine \( k \), the relative pheromone trail between them is updated by Eq. (43) and (45). Otherwise, the relative pheromone trail is obtained by Eq. (44).

\[ \xi_{kl'i'j'}(tn + 1) = \xi_{kl'i'j'}(tn + 1) + \Delta \xi_{kl'i'j'}(best) \]

\[ \forall (i', j'), (i', j'), k, x, O_{i'j'} \in Q_k^{(x)}(s), O_{i'j'} \in Q_k^{(x)}(s + 1) \]

\[ \xi_{kl'i'j'}(tn + 1) = \rho \cdot \xi_{kl'i'j'}(tn) \]

The planning horizon was divided into equal frozen horizons. The scheduling assigned machines in order of priority to each operation of parts families on each frozen horizon. If all of production batches were processed in a normal period of time, the problem would be solved; otherwise, changing the shift in order to assign the production batches on each frozen horizon would be done. After this step, if there was no enough time to assign the operation to the machines in this step, the problem would be continued with enlarging the frozen horizon and determining the size of production batches on each new frozen horizon again (see Figure 3).
5. Case Study

To examine the proposed approach, a job center in Safe Sanat Company was investigated. The company, founded in 1992, is a supplier of the automotive industry in Iran and is expert in producing side door locks.

To produce thirteen different products, eighty-six parts in the form of sixty-four families were processed in the job center. Parts in each family had a similar process. Also, each family consisted of 1 to 5 different parts.

The production batch consisted of \( K \) lots related to a certain parts family. Each lot pertained to an identified part. All lots were stacked to reach an equal size. Each time all parts were processed by a certain machine, and then they were added to their lots in order to reach the specified quantity.

The job center had fourteen machines. These machines consisted of hydraulic and kick press. The normal time for each machine in shifts (1) and (2) was seven hours and a quarter. Shift (3) was also considered as extra time for each machine with the same normal time.

The current condition for production planning and scheduling in this job center involved determining the lot size for each part on the planning horizons. Then parts families were produced only based on experiences and without respect to the optimal production batch, Make-span and time-limit for each kind of machine. In this situation, there was fluctuation in the production and the end inventory of the parts. Therefore, there were a lot of parts more or less than they were needed at the end of planning horizon.

In order to improve the planning at this work center, alternative routes for each operation, if it was possible, were considered. Corresponding to each family, there was a production batch (\( \text{PB}_{i,j,s} \)) in every frozen horizon, in case it was produced. The production batch for each family was considered equal on any frozen horizon. Also, the number of operations for each family was between 1 to 5.

Safety stocks were calculated for each part based on the planning horizon, lead-time, and 85% significance level in a normal distribution.

6. Results of the Proposed Production Planning and Scheduling

6.1. The production lead-time and external safety lead-time

In order to calculate the safety stock, production lead-time and external safety lead-time of each part are calculated as follows by (52):

\[
\overline{L}_{ik} = \sum_{j=1}^{n} (\overline{FD}_{jk} + \overline{S}_{ij} + \overline{UPT}_{ijr} + \overline{W}_{rs}(n')) + \frac{n_i}{\text{Day Per Month}}
\]

where

- \( \overline{L}_{ik} \): mean production lead-time for the part \( k \) of family \( i \)
- \( \overline{FD}_{jk} \): unit production time of route \( r \) for operation \( j \) of family \( i \)
- \( \overline{S}_{ij} \): mean setup time for operation \( j \) of family \( i \)
- \( \overline{UPT}_{ijr} \): for a different route
- \( \overline{W}_{rs}(n') \): normal production time for shift \( s \) in a day (in seconds)
- \( n' \): average forecasted demand for the part \( k \) of family \( j \) during planning horizon
- \( n_i \): the number of initial frozen horizon
- \( n_i \): the number of internal operation for family \( i \)

The external safety lead-time for each part is also calculated by Eq. (53):

\[
\overline{E}_{ik} = \frac{n_i}{\text{Day Per Month}}
\]

Where
\( L_{ik} \): mean external safety lead-time for the park \( k \) of family \( i \) (in month)

\( n_i' \): the number of external operation for family \( i \)

6.2. Results of the proposed framework for part families

Table A-1 in the appendix presents the forecasted demand \((FD_{jk})\), safety stock \((SS_{jk})\) and Temporary lot size \((TLS_{jk})\) for each part. Table A-2 in the appendix shows the actual lot size \((ALS_i)\), Actual start of Production \((ASOP_i)\), and Production Batch \((PB_{BH_i})\) for each family.

6.3. Results of the proposed approach for scheduling on optimal frozen horizons

\( PHSD \) was January 1\(^{\text{st}}\) 2012 and \( PHFD \) was January 30\(^{\text{th}}\) 2012. Our \( FH \) consisted of six, ten, fifteen, and thirteen days. The non-working days were January 6\(^{\text{th}}\) 2012, January 13\(^{\text{th}}\) 2012, January 20\(^{\text{th}}\) 2012, January 21\(^{\text{st}}\) 2012, and January 27\(^{\text{th}}\) 2012. Our case study had sixty seven families with the maximum of five operations at each family. The number of machines was fourteen.

This case study was a large size problem. Therefore, the time to reach the final answer was directly related to increasing the number of ants and their iterations. The number of ants was 50 \((n=50)\) and the number of iterations was 60 \((n=60)\). Parameters for scheduling were \( \rho = 0.9, \omega = 2, \gamma = 1, \beta = 2, a = 1 \). Moreover, \( \tau_{\text{max}}(1) = \xi_{\text{max}}(1) = 0.1 \). The time for solving was between two and a half hours and three hours. The \( f(\text{best}) \) is also calculated as follows.

For each \( tn \) between the interval \([1,100]\), the \( f(\text{best}) \) is equivalent to the \( f(\text{best}) \). For each \( tn \) between the interval \([100, 200]\), after every four iterations, we used the \( f(\text{best}) \) instead of the \( f(\text{best}) \). For each \( tn \) between the interval \([200, 300]\), after every three iterations, we replaced the \( f(\text{best}) \) instead of the \( f(\text{best}) \). For each \( tn \) between the interval \([300,400]\), after every two iterations, we used the \( f(\text{best}) \) instead of the \( f(\text{best}) \). For each \( tn \) between the interval \([400,500]\), after every iteration, the \( f(\text{best}) \) is equivalent to the \( f(\text{best}) \). Finally, for \((500 < tn)\), after every iteration, we replaced only the \( f(\text{best}) \) instead of the \( f(\text{best}) \).
6.4. Comparing the results of ant colony optimization with the mixed integer programming

Table 1 shows that results of the proposed ACO were near the optimal solution. Indexes used for this comparing are $D_1$ and $D_2$. The $D_1$ was equivalent to the deviation between the best result of the proposed meta-heuristic and the lower bound of the Lindo Software for the same problem. The $D_2$ was equivalent to the deviation between the results of the proposed meta-heuristics that is used by (54). The sample size of each problem was also (13).

Figure 4 depicts the $D_2$ for different problems. Comparing the CPU time for different methods was also done in the figure 5. Therefore, we conclude that the proposed algorithm has efficiency to minimize the Make-span in FJS.

$$D_2 = \frac{\sum (f - f^*)}{n f^*}$$

(54)

where $f^*$ the best results from the proposed algorithm in the specified sample

Table 1
Comparing the results of ant colony optimization with the mixed integer linear programming

<table>
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<tr>
<th>Row</th>
<th>J</th>
<th>M</th>
<th>Opr</th>
<th>Result of Lindo Software</th>
<th>Result of Proposed Algorithm</th>
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<th>$D_2$</th>
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<td>CPU</td>
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<td>660</td>
</tr>
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</table>

J: number of family; M: number of machines; Opr: number of operation

Fig. 4. Index $D_2$ for different problems
Besides, in order to survey the proposed algorithm, we used t statistic with the infinity upper bound. The mean for this hypothesis test is obtained by the Lindo Software. Table2 indicates the results of the hypothesis test for different problems. The sample size is 13 (N=13) and the confidence level is 95%.

6.5. Final results of production scheduling on the frozen horizons

The results of scheduling on the frozen horizons are presented in TableA-3 in the appendix. The make-span is also presented in the table 3. In addition, the optimal shifts are presented in Tables 4, 5, and 6.

6.6. The parts inventory of the proposed framework at the end of planning horizon and evaluating its effectiveness

The number of different parts in each family at the end of planning horizon by the proposed framework was calculated by Eq. (55). Also, Eq. (56) calculated the efficiency of the proposed framework at the end of planning horizon:

Table 2
A statistical comparison between the results of the proposed algorithm and LINDO results

Table 3
Make-span for the optimal frozen horizon

Table 4
Shifts for each machine in the frozen horizon (10-1)
Table 5
Shifts for each machine in the frozen horizon (10-2)

<table>
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<th>Machine number</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
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<th>M6</th>
<th>M7</th>
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<th>M9</th>
<th>M10</th>
<th>M11</th>
<th>M12</th>
<th>M13</th>
<th>M14</th>
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<td>1</td>
<td>1</td>
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Table 6
Shifts for each machine in the frozen horizon (10-3)

<table>
<thead>
<tr>
<th>Machine number</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
<th>M7</th>
<th>M8</th>
<th>M9</th>
<th>M10</th>
<th>M11</th>
<th>M12</th>
<th>M13</th>
<th>M14</th>
</tr>
</thead>
<tbody>
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<td>3</td>
<td>3</td>
<td>3</td>
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<td>2</td>
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<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
I'_{jk} = ALS_j - TIS_{jk} \tag{55}
\]

\[
\Delta I = I'_{jk} - I''_{jk} \tag{56}
\]

where

\[I'_{jk}\] inventory for the part k of family j at the end of planning horizon that is calculated by the operator

\[I''_{jk}\] inventory for the part k of family j at the end of planning horizon that is calculated by the proposed framework

Table A-4 in the appendix shows the \[\Delta I\] for different parts families. This indicator indicates a more decrease in the parts inventory at the end of planning horizon than the traditional method.

We coded this problem in Visual C#. We also designed a database in SQL Server (2000). A PC with Core 2 Duo CPU, a 2.53GHZ processor and 4 GIG Ram was used for running the problem.

7. Future Research

A hierarchical framework for production planning and scheduling in FJS with respect to priority for production families was presented in this paper. This approach resulted in more decrease in the parts inventory at the end of planning horizon than the traditional method. This decrease led to the falling cost of material and human resources. Moreover, the utilization of the machines with this framework was increased. Finally, the results of the proposed scheduling algorithm were near the optimal solution.

With regard to future research, we recommend that researchers may investigate scheduling algorithms where production batches can be processed concurrently on identical machines. Also, investigations on presenting an MILP model, in a way that machines can be assigned to operations in order of priority, are more desirable.

8. References

### Appendix

Table A-1
The calculated parameters for each part

| K  | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| J  | 1   | 2   | 2   | 3   | 4   | 4   | 4   | 5   | 6   | 7   | 8   | 9   | 9   | 10  | 10  | 11  | 11  | 12  | 12  | 13  | 13  | 13  | 13  | 13  |
| FDjk | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 40000 | 40000 | 40000 | 40000 | 40000 | 40000 | 40000 | 40000 | 40000 | 40000 | 40000 | 40000 | 40000 | 20000 | 20000 |
| SSjk | 4210  | 5351 | 3698 | 3698 | 3794 | 4807 | 3239 | 3239 | 8537 | 6687 | 8242 | 14565 | 8564 | 6776 | 7895 | 6773 | 7890 | 8471 | 6666 | 3930 | 4981 |
| TLSjk | 15325 | 6377 | 15928 | 17168 | 19652 | 20792 | 18360 | 48537 | 40687 | 42503 | 82565 | 41044 | 45091 | 16624 | 42695 | 29323 | 8471 | 24700 | 21750 | 23510 |
| K  | 24  | 25  | 26  | 27  | 28  | 29  | 30  | 31  | 32  | 33  | 34  | 35  | 36  | 37  | 38  | 39  | 40  | 41  | 42  | 43  | 44  | 45  |
| J  | 15  | 16  | 16  | 17  | 17  | 18  | 19  | 19  | 19  | 20  | 20  | 21  | 21  | 22  | 22  | 22  | 22  | 22  | 22  | 22  | 22  | 22  |
| FDjk | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 | 20000 |
| SSjk | 4798  | 3788 | 3811 | 4813 | 3594 | 4539 | 8410 | 3809 | 4826 | 3195 | 3195 | 3175 | 3175 | 3212 | 3212 | 8176 | 6450 | 6450 | 5543 | 5240 | 527  | 5240 |
| TLSjk | 6727  | 10315 | 46996 | 9646 | 8636 | 147 | 5281 | 23809 | 24826 | 3999 | 13612 | 6508 | 8498 | 9087 | 10727 | 40176 | 46450 | 20407 | 17183 | 5449 | 1658 | 2870 |
| K  | 47  | 48  | 49  | 50  | 51  | 52  | 53  | 54  | 55  | 56  | 57  | 58  | 59  | 60  | 61  | 62  | 63  | 64  | 65  | 66  | 67  | 68  |
| J  | 27  | 28  | 29  | 30  | 31  | 32  | 33  | 34  | 34  | 35  | 36  | 37  | 38  | 39  | 40  | 41  | 42  | 43  | 44  | 45  | 46  |
| FDjk | 8640  | 8640 | 8640 | 8640 | 8640 | 8640 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 |
| TLSjk | 7894  | 9598 | -1369 | 10644 | -1303 | -1225 | 9479 | 13239 | 13988 | 13029 | 12287 | 4480 | 2520 | 1411 | 6936 | 1916 | 1823 | -7753 | 5937 | -5066 | 43351 | 11967 |
| K  | 70  | 71  | 72  | 73  | 74  | 75  | 76  | 78  | 79  | 80  | 81  | 82  | 83  | 84  | 85  | 86  | 87  | 88  | 89  |
| J  | 48  | 49  | 50  | 51  | 52  | 53  | 55  | 57  | 58  | 59  | 60  | 61  | 62  | 63  | 64  | 66  | 67  |
| FDjk | 5000  | 5000 | 5000 | 5000 | 5000 | 30000 | 15000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 2400 | 1200 | 1200 | 1200 | 1200 |
| SSjk | 1749  | 1929 | 1880 | 1975 | 2100 | 2692 | 1852 | 305  | 291  | 295  | 311  | 303  | 329  | 286  | 286  | 285  | 295  |
| TLSjk | -7117 | 6929 | -248 | 6975 | 7100 | 17882 | 4855 | -1315 | 1491 | 1000 | -679 | -1597 | -7117 | -1138 | -3287 | -845 | -308 |

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### Table A-2
Calculated parameters for each family

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<th>Row</th>
<th>J</th>
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<th>ASOPj</th>
<th>Optimized Frozen Horizon (Day)</th>
<th>Production Batch Per Frozen Horizon (PB_{\text{F,\text{J}}})</th>
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*Table A-2 (CONTINUED) Calculated parameters for each family*
The result of production scheduling for each machine on each frozen horizon

Frozen Horizon = 10

N = 1

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Frozen Horizon = 10

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sequence of operations for machine (6) o371 o581 o582 o531 o332 o372 o374

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sequence of operations for machine (7) o401 o341 o342 o192 o162 o261 o402 o71 o403 o252 o24

Table 19

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sequence of operations for machine (8) o412 o133 o382

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sequence of operations for machine (9) o391 o351 o411 o134 o12 o375 o452

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sequence of operations for machine (10) o461 o191 o373 sequence of operations for machine (11) o431

Table 22

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sequence of operations for machine (12) o461 o191 o373 sequence of operations for machine (13) o431 o432 sequence of operations (14) o431

Frozen Horizon = 10

N = 3

Table 23

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(sequence of operations for machine (1) o361 o11 o91 o511 o111 o21 o491

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sequence of operations for machine (2) o451 o201 o131 o591 o221 o531 o631 o281 o171 o521 o592 o522 o523
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Table 4
\(\Delta I\) For different parts families