A New Fuzzy Method for Assessing Six Sigma Measures

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Abstract

Six-Sigma has some measures which measure performance characteristics related to a process. In most of the traditional methods, exact estimation is used to assess these measures and to utilize them in practice. In this paper, to estimate some of these measures, including Defects per Million Opportunities (DPMO), Defects per Opportunity (DPO), Defects per unit (DPU) and Yield, a new algorithm based on Buckley's estimation approach is introduced. The algorithm uses a family of confidence intervals to estimate the mentioned measures. The final results of introduced algorithm for different measures are triangular shaped fuzzy numbers. Finally, since DPMO, as one of the most useful measures in Six-Sigma, should be consistent with costumer need, this paper introduces a new fuzzy method to check this consistency. The method compares estimated DPMO with fuzzy customer need. Numerical examples are given to show the performance of the method.

Keywords: Six Sigma; Fuzzy set; Fuzzy estimation; DPU; DPO; Yield; DPMO.

1. Introduction

Six-Sigma as a comprehensive and flexible system was developed by Bill Smith at Motorola in 1980s and is used for achieving, sustaining and maximizing business success. Materials of Six-Sigma are close understanding of customer needs, disciplined use of facts, data, and statistical analysis, and diligent attention to managing, improving, and reinventing business processes (Brefogle, 1964). Six-Sigma to improve its targets looks at them as different processes. Then it analyses, controls and improves these processes through different measures. Some of these measures are Capability of process (Cp, Cpk), Defects per Million Opportunities (DPMO), Defects per Opportunity (DPO), Defects per unit (DPU) and Yield, for process analyzing and capability of the gauge (Cg, Cgk) for gauge analyzing and so on. This paper examines DPU, DPO, DPMO and Yield in a fuzzy environment.

In Six-Sigma, as an important sub title of quality issue (as a whole), classifying of measures depends on type of the data they are due to. Generally, data for quality control purposes are collected by observation. These data are classified as either variables or attributes. Variables are those quality variables which are measurable. There are two types of variables: 1) continuous variables, 2) discrete variables.

Continuous variables are those variables which are capable of any degree of subdivision (Brefogle, 1964). Meters and liters are examples of continuous data and normal distribution is usually used for (direct fitting or estimated fitting of) these type of data (Brefogle, 1964). In the literature of Six-Sigma as a sub-title of quality (as a whole issue) there are different measures of this class. C_p, C_{pk}, and C_{pm} are practical measures of this class that reflecting the capability of a process. These measures are

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studied in different papers such as (Chen, 2008). Among the aforementioned papers (Chen., K. S & Chen, T. W, 2008; Parchami & Mashinchi, 2007) there are some papers that study these measures in fuzzy environments. (Parchami & Mashinchi, 2007) specifically used Buckley's approach for their fuzzy estimation.

Discrete variables are those that exhibit gaps (Brefogle, 1964). Number of defective rivets in a truck trailer (which can be any whole number 0, 1, 2…) is example of discrete data. Poisson distribution is usually used for these types of data (Brefogle, 1964). Generally, continuous variables are measurable while discrete data are countable.

Attributes are usually known as those types of data which are judged by visual observation. The answer is wrong or true, the switches are on or off are examples of this class. Binomial distribution is usually used for this class.

In order to determine differences among Binomial capability and Poisson capability, it should be mentioned that if we have one opportunity for evolving of defect per a unit these two types of capability treat approximately the same. However, the advantage of Poisson capability is that it can be used in a situation with more amounts of opportunities for evolving of defect per a unit (Brefogle, 1964).

Measures that Poisson distribution or Poisson capability can be used for them are DPU, DPO and DPMO. This paper focused on these measures (especially on DPMO) and makes them fuzzy. Meanwhile, since Yield is related with previous measures in its nature (as a process measuring index) and calculations, it is also studied as a fuzzy measure.

Buckley, in his papers (Buckley, 2005), has defined $(1-\beta)100\%$ confidence intervals for a parameter as a family of $\alpha-cuts$ of a triangular shaped fuzzy number.

In this paper his approach is applied to find fuzzy estimates of DPMO, DPO, DPU and Yield. Estimating of sigma as it is clear from the name of Six-Sigma is a crucial issue. To estimate sigma, Six-Sigma uses different type of measures and DPMO is the most practical one. If we can implement Six-Sigma effectively, it results in at most 3.4 defects per million opportunities. After calculating DPMO, we can translate it to sigma by means of a handy table like Table 1.

Then, since DPMO should be consistent with customer need, this paper, to complete assessing of DPMO in fuzzy environment, introduces a fuzzy comparing method that uses Buckley's hypostasis testing idea (Buckley, 2005) with some adaption to make calculations easier for this especial study. Buckley in his paper introduced a general method to compare fuzzy statistic with fuzzy critical value in a hypothesis testing.

Table 1
Conversion of DPMO to Sigma

DPMO (Defect per Million Opportunities)	Sigma Quality level
308,537	2
66,807	3
6,210	4
233	5
3.4	6

The organization of this paper is as follows. Section 2 reviews the classical Six Sigma and DPMO. Section 3 discusses Buckley's approach and presents the $\alpha-cuts$ of fuzzy estimation for DPMO. Section 4, using Buckley's approach, proposes a new algorithm for fuzzy estimation of DPMO based on predefined $\alpha-cuts$ and illustrates it by two examples. Section 5, presents a method for comparing the fuzzy estimated DPMO with a fuzzy number which presents customer's specification and a numerical example is given to illustrate the method. Section 6 concludes the paper.

2. Classical Six-Sigma

Each business is like a child who is learning how to ride his/her bike, and we as parents (business owners) are there to help and offer encouragement. We want to see the kid succeeds and the system owner wants to see its offspring thrive. For beginning we give the kid a push and for a while s/he rides beautifully, balanced, head and erect. "Look I'm doing" is what we hear just before the kid runs off and runs into a bush. Of course we know the only way to learn biking is falling off and running into the bushes. Likewise, companies work but they should get back on the path fast enough and try again. Now, what both successful bike riding and business managing are due, is a closed loop system in which both internal and external types of information (feedback) tell the rider/manager how to correct course, steer successfully (Brefogle, 1964).

In order to create a closed loop system, Six-Sigma looks at its target as a process and defines different types of measures. These measures cause Six-Sigma to become enough sensitive to reduce the company's wobbling and keeps it safely on the path of success. In Six-Sigma's vocabulary, the wobbling of a business is translated as variation and bad variation which have negative impacts on customers called defect. Management by looking at variation can get a fuller understanding of their process performance (Brefogle, 1964). Finally, what Six-Sigma is looking for is to reduce and narrow variation to such a degree that six sigma (or standard deviation) can be squeezed within the limits defined by the customer specification (or need) like Fig. 1.

In Six-Sigma, we should first clearly define what the customer wants as an explicit requirement. Then, we should count the measures of our system to recognize the quality of our performance in related to customer satisfaction.

Mostly mentioned measures are due to defects. Therefore, in the rest of this section different definitions and formulation of measures which are studied in the paper are introduce and the relationship among them is discussed.

A defect is any instance or event in which the product or process fails to meet a customer requirement (Brefogle, 1964).

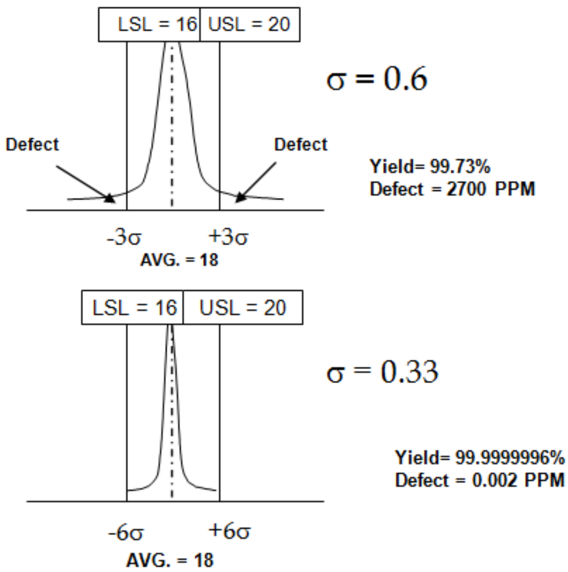

Fig. 1.The distinctive impact of Six Sigma on process performance

Counting defects, we can calculate the "yield" of the process (percentage of items without defects), and then calculate the DPMO and finally use a handy table like Table 1 to determine the Sigma level. Sigma levels of performance are often expressed in Defects per Million Opportunities or DPMO. DPMO simply indicates how many errors would show up if an activity were to be repeated a million times (Brefogle, 1964). DPMO has two types named discrete and continuous which are defined as follows:

Continuous DPMO is the one that can be measured on an infinity-divisible scale or continuum: e.g., weight, time, height, temperature, ohms.

A discrete DPMO is anything else that doesn't fit the criteria for "continuous." Discrete items might include characteristics or attributes, count of individual items, etc."

and recognized as Eq.1.

$$DPMO = (1 - y_{rt}) * 1000000 \tag{1}$$

Where

y_{rt} = Yield: the area under the probability density curve between tolerances

Yield in Discrete DPMO is proven from Poisson distribution as Eq.2.

$$y_{rt} = yield = p(X = o) = \frac{e^{-\lambda}\lambda^{x}}{x!} = e^{-\lambda} = e^{-dpo} \tag{2}$$

and continuous DPMO is proven from normal distribution as:

$$y_{rt} = yield = p(X \geq a) = p(\frac{X - \mu}{\sigma} \geq \frac{a - \mu}{\sigma}) = p(Z \geq z) \tag{3}$$

It should be noticed that what this paper discusses is discrete DPMO.

References use point estimates to estimate DPO, yield and DPMO, nevertheless, point estimate can't account the uncertainty due to sampling variability properly. Therefore, we can't make good inferences about the true value of DPMO. So this paper to guard against uncertainty, uses confidence intervals and builds a new algorithm which not only covers the characteristics of point estimates but also interval estimates.

3. Buckley's Estimation Approach

In what follows, with modification, fuzzy estimation based on Buckley's approach (Buckley, 2005) is presented. First some notation is introduced. A triangular shaped fuzzy number N is a fuzzy subset of the real numbers R satisfying:

(i) N(x) =1 for exactly one x $\in$ R

(ii) For $\alpha \in (0,1]$, the $\alpha - cut$ of N is a closed and bounded interval, which I denote by $N_\alpha = [n_1(\alpha), n_2(\alpha)]$, where $n_1(\alpha)$ is increasing and $n_2(\alpha)$ is decreasing continuous functions.

Triangular shaped fuzzy number is used for parameter estimation. Let X be a random variable with p. d.f. (p.m.f .) $f(x;\theta)$ for single parameter θ. Assume that θ is unknown and must be estimated from a random sample $X_1, X_2, X_3, ..., X_n$. Let y=u($X_1, X_2, X_3, ..., X_n$) be a statistic used to estimate θ.Given the values of these random variable , e.g. $X_i = x_i$, $1 \le i \le n$,we can obtain a point estimate. $\hat{\theta} = y = u(x_1, x_2, x_3, ..., x_n)$ for θ. Since we never expect this point estimate be exactly equal to θ, we often also compute a $(1-\beta)100\%$ confidence interval for θ.

This paper denotes a $(1-\beta)100\%$ confidence interval for θ by $[\theta_1(\beta), \theta_2(\beta)]$, for $0 < \beta < 1$.Thus the interval $\theta_1 = [\hat{\theta}, \hat{\theta}]$ is 0% confidence interval for θ and $\theta_0 = \ddot{\theta}$ is a 100% confidence interval for θ, where $\ddot{\theta}$ is the whole parameter space. Then we have a family of $(1-\beta)100\%$ confidence intervals for θ ,where $0 \le \beta \le 1$ .If we place these confidence intervals, one on top of the other, we obtain a triangular shaped fuzzy number θ whose $\alpha - cuts$ are the following confidence intervals:

$\theta_\alpha = [\theta_1(\alpha), \theta_2(\alpha)]$ for $0 < \alpha < 1 : \theta_0 = \ddot{\theta}$ and $\theta_1 = [\hat{\theta}, \hat{\theta}]$.

Hence we use more information about θ rather than a point estimate, or just a single interval estimate. It is easy to generalize Buckley's method in the case where θ is a vector of parameters (Buckley, 2005).

It should be mentioned that this paper uses β here since α, usually employed for confidence intervals, is reserved for θ, $\alpha - cuts$ of fuzzy numbers . The rest of the section computes $(1-\beta)100\%$ confidence intervals for DPO, yield and DPMO. In section 4 these intervals are used as $\alpha - cuts$ of the fuzzy estimators of DPMO.

3.1. Cuts of a fuzzy estimate for DPO

According to normal approximation to the Poisson we can obtain following confidence intervals for parameter of Poisson distribution named $\lambda = DPO$ for a determined level of β :

$$\{dpo - Z_{\frac{\alpha}{2}}\sqrt{\frac{dpo}{n}}, dpo + Z_{\frac{\alpha}{2}}\sqrt{\frac{dpo}{n}}\} \text{ for } 0<\alpha<1. \quad (4)$$

Since most of the time λ is unknown and we should estimate it by sampling and also since according to MLE approach the point estimator of λ is $\bar{X} = \overline{dpo}$, we had better use $\overline{dpo}$ instead of DPO as follows:

$$\{\overline{dpo} - Z_{\frac{\alpha}{2}}\sqrt{\frac{\overline{dpo}}{n}}, \overline{dpo} + Z_{\frac{\alpha}{2}}\sqrt{\frac{\overline{dpo}}{n}}\} \text{ for } 0<\alpha<1 . \quad (5)$$

3.2. Cuts of a fuzzy estimate for yield

To have a left-right fuzzy number which represents fuzzy yield (discrete one) confidence interval of DPO is used as follows:

$$\{e^{-(\overline{dpo} - Z_{\frac{\alpha}{2}}\sqrt{\frac{\overline{dpo}}{n}})}, e^{-(\overline{dpo} + Z_{\frac{\alpha}{2}}\sqrt{\frac{\overline{dpo}}{n}})}\} \text{ for } 0<\alpha<1. \quad (6)$$

and the examples, which give in the next section, show how it creates a fuzzy number.

3.3. Cuts of a fuzzy estimate for DPMO

Now according to what was mentioned in this section and previous section (equation 2) we can define $\alpha - cuts$ for discrete DPMO as follows:

$$\{(1-(e^{-(\overline{dpo} - Z_{\frac{\alpha}{2}}\sqrt{\frac{\overline{dpo}}{n}})}))*1000000, (1-(e^{-(\overline{dpo} + Z_{\frac{\alpha}{2}}\sqrt{\frac{\overline{dpo}}{n}})}))*1000000\} \text{ for } 0<\alpha<1 . \quad (7)$$

4. A New Algorithm to Estimate Fuzzy DPO, Yield and DPMO

Let θ_α^i for $\alpha \in (0,1)$ and i=1, 2, 3 be as in section 3. In the next part Buckley's approach is extended to create a new algorithm to find fuzzy estimates for DPO, yield and discrete DPMO.

4.1 FESSM Algorithm

(1) Let $\theta_0^i = R^+$, $\theta_1^i = [\hat{\theta}^i, \hat{\theta}^i]$ for i=1,2,3 $\hat{\theta}^1 = dpo$, $\hat{\theta}^2 = yield$, $\hat{\theta}^3 = dpmo$ and R^+ is the set of all positive real numbers.

(2) Place θ_α^i; $0 \le \alpha \le 1$, one on top of the other , to produce a triangular shaped fuzzy number $\hat{\theta}^i$ for i=1,2,3; where $\hat{\theta}^1 = dpo$, $\hat{\theta}^2 = yield$, $\hat{\theta}^3 = dpmo$.

Since DPO, yield and DPMO are generally unknown and must be estimated from observations, uncertainty due to sampling variability is unavoidable. Therefore this paper introduces FESSM algorithm to guard against uncertainty in order to get close to the real value of the measures. In what follows the performance of FESSM algorithm is illustrated by two examples.

E**xample 1.** Consider that information of producing 1000 shafts (in a month) is as follows:

1-oprator: inspects 1 shaft per 10 produced shafts, so in last month (or per 1000 shafts), he has inspected 100 shafts. He has founded 12 defects in his inspection.
2-quality control inspector: has taken 100 samples and founded 3 defects. Therefore, we have the following results:

Lot size: 1000
Sample size: 100+100=200
Number of defects: 12+3=15

So in a classical calculation DPO, yield and DPMO are computed as follows Eq.8 to Eq.10.

$$\text{DPO} = \frac{\text{Number of defects}}{\text{Number of opportunities}} = \frac{15}{200} = 0.075 \tag{8}$$

$$\text{yield} = e^{-\text{dpo}} = e^{-0.075} = 0.9277 \tag{9}$$

$$\text{DPMO} = (1-\text{yield})*1000000 = (1-.9277)*1000000 = 72256.51 \approx 72257 \tag{10}$$

Now, according to what was defined in (5), (6) and (7) we can compute $\alpha - cuts$ of $\hat{\theta}^i$ for *i*=1,2,3 as follows:

$$\hat{\theta}^1 = \{0.075 - Z_{\frac{\alpha}{2}}\sqrt{\frac{0.075}{200}}, 0.075 + Z_{\frac{\alpha}{2}}\sqrt{\frac{0.075}{200}}\}$$

for $0 \le \alpha \le 1$. (11)

$$\hat{\theta}^2 = \{e^{-(0.075 - Z_{\frac{\alpha}{2}}\sqrt{\frac{0.075}{200}})}, e^{-(0.075 + Z_{\frac{\alpha}{2}}\sqrt{\frac{0.075}{200}})}\} \text{ for}$$

$0 \le \alpha \le 1$. (12)

$$\hat{\theta}^3 = \{(1-(e^{-(0.075 - Z_{\frac{\alpha}{2}}\sqrt{\frac{0.075}{200}})}))* 1000000, (1-(e^{-(0.075 + Z_{\frac{\alpha}{2}}\sqrt{\frac{0.075}{200}})}))*1000000\}$$

for $0 \le \alpha \le 1$. (13)

In the first step of FESSM algorithm, we can obtain θ_0^i and θ_1^i for *i*=1, 2, 3(for each i separately). During the second step, by placing $\theta_0^i, \theta_\alpha^i$ for $\alpha \in (0,1)$, and θ_1^i for *i*=1, 2, 3(for each *i* separately), which are calculated by the first step and (5), (6) and (7) respectively, one on top of the other, we can obtain fuzzy estimates for DPO, yield and DPMO respectively. The graphs of their membership functions are shown in Fig. 2, by Minitab software (it should be mentioned that first for $0 \le \alpha \le 1$ different calculations have been done in Excel software, then to figure fuzzy numbers more effective data have been transferred to Minitab software). Note that in the classical method, as it was shown in (8), (9) and (10), one can find estimates $\overline{dpo} = 0.075$, $yield = .9277$ and $dpmo = 72256.51$. We would never expect this precise point estimate to be exactly equal to the parameter value, so we often compute $(1-\beta)100\%$ confidence intervals for our parameters. The fuzzy estimate obtained by the FESSM algorithm contains more information than a point or interval estimate, in the sense that the fuzzy estimate contains point estimates and $(1-\beta)100\%$ confidence intervals for all at once for $\beta \in [0,1)$, which is very useful for a practitioner.

From the fuzzy estimate, one can conclude that the classical estimate $dpmo = 72256.51 \approx 72257$ belongs to the fuzzy estimate DPMO with grade of membership equal to 1. It is obvious that fuzzy set of DPMO contains more elements other than "72256.51" with corresponding grades of membership. For example, one can say that $dpmo = 69996.17$ belongs to the DPMO with grade of membership $dpmo(69996.17) = 0.9$ (Fig. 2). Similarly, we can see that $dpo = 0.0701$ belongs to the DPO with grades of membership $dpo(0.0701) = 0.8$.

Example 2. In Example 1, we considered three fuzzy estimates of DPMO based on different samples from the same process. By the FESSM algorithm, the fuzzy estimates of DPMO obtained for the three samples, are depicted in Fig. 3.
Classical answers are:

For n=100, $dpo = 0.15$, $dpmo = 139292.0$

For n=200, $dpo = 0.075$, $dpmo = 72256.51 \approx 72257$

For n=300, $dpo = 0.05$, $dpmo = 48770.6$

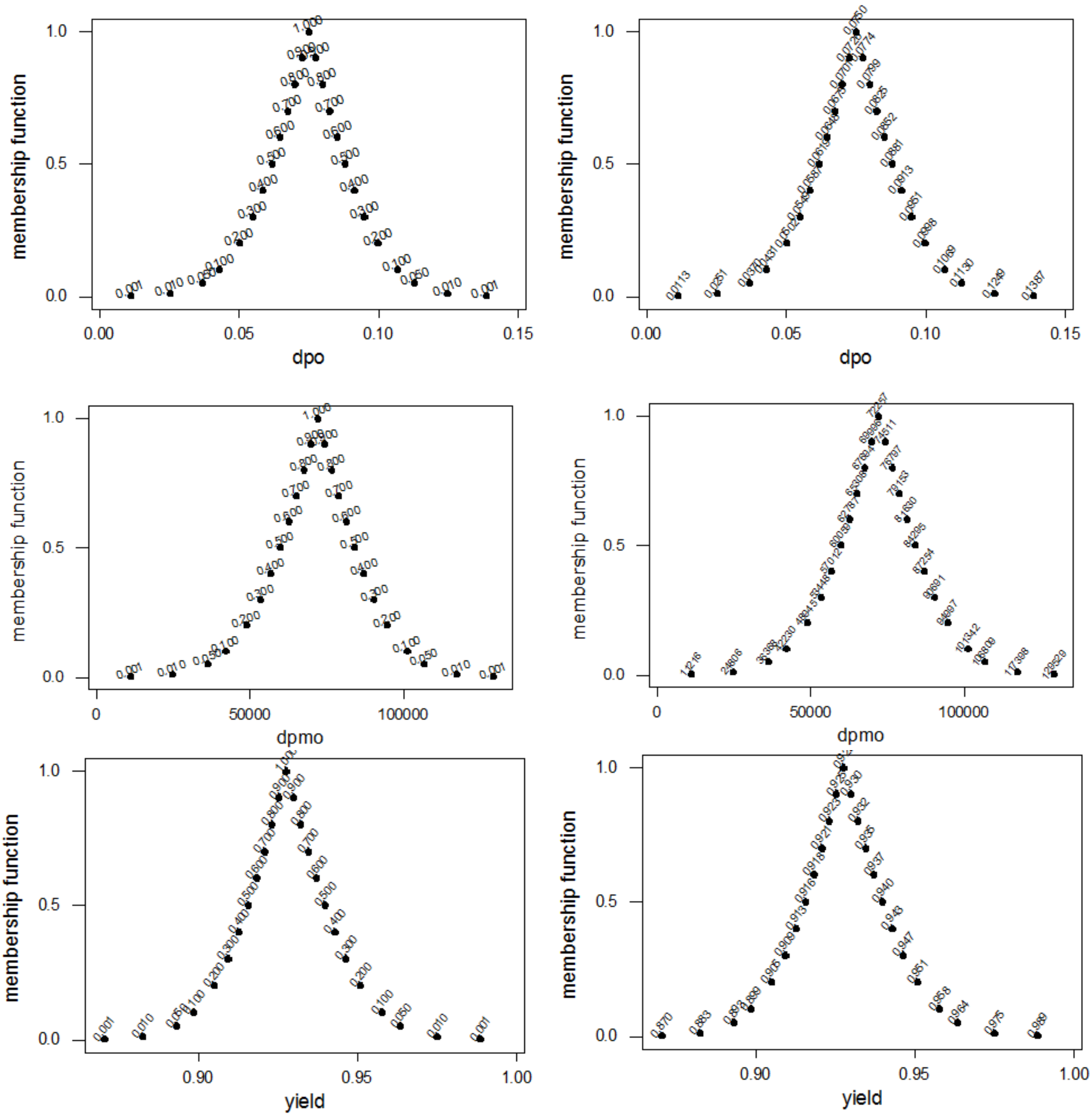

Fig. 2. Fuzzy membership function of DPO, yield, DPMO

For n= 100, $(dpmo)_{\alpha} = [(1-(e^{-(0.1500-Z_{\frac{\alpha}{2}}\sqrt{\frac{0.1500}{100}})}))*1000000, (1-(e^{-(0.1500+Z_{\frac{\alpha}{2}}\sqrt{\frac{0.1500}{100}})}))*1000000]$ for $0 \le \alpha \le 1$. (14)

For n= 200, $(dpmo)_{\alpha} = [(1-(e^{-(0.075-Z_{\frac{\alpha}{2}}\sqrt{\frac{0.075}{200}})}))*1000000, (1-(e^{-(0.075+Z_{\frac{\alpha}{2}}\sqrt{\frac{0.075}{200}})}))*1000000]$ for $0 \le \alpha \le 1$. (15)

For n= 300, $(dpmo)_{\alpha} = [(1-(e^{-(0.0500-Z_{\frac{\alpha}{2}}\sqrt{\frac{0.0500}{300}})}))*1000000, (1-(e^{-(0.0500+Z_{\frac{\alpha}{2}}\sqrt{\frac{0.0500}{300}})}))*1000000]$ for $0 \le \alpha \le 1$. (16)

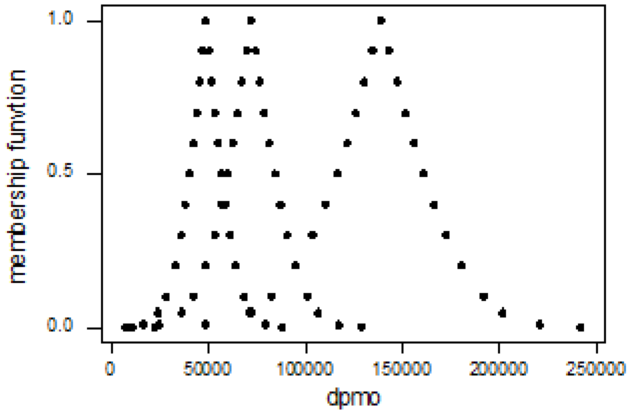

Fig. 3.The larger sample size (n=100,200,300), the less DPMO and the less fuzziness.

Remark 1. As seen in Example 2, increasing the sample size (n) not only decreases the amount of the DPMO but also its fuzziness, therefore it leads to sharper and smaller triangular shaped fuzzy numbers.

5. New Method for Comparing Fuzzy DPMO with Fuzzy Customer Need

Consider our customer gives us a fuzzy number as critical value for DPMO or customer need. We now have a fuzzy number for DPMO (Fig. 4 and 16) and a fuzzy number for customer need (Figs. 6 and 17). Our final decision will depend on the relationship between fuzzy DPMO and fuzzy customer need. This can be best explained through studying Fig. 5. Figure 5 illustrates our final decision rule: reject or do not reject. In fact, the fuzzy number for DPMO should be triangular shaped fuzzy number, like in Fig. 3, instead of triangular fuzzy numbers, but in order to simplify the calculating this paper uses estimated fuzzy DPMO like the one you see in Fig. 4 and membership function as (17).

As you can see in Fig. 4 the estimation is pessimistic and cause more fuzziness. Therefore the new method which this section introduced is strict. The next paragraph explains the new strict method to compare fuzzy DPMO with fuzzy customer need.

$$\begin{cases} (x-45439)/(72257-45439), & 45439 \le x \le 72257 \\ 1, & x=72257 \\ (98321-x)/(98321-72257), & 72257 \le x \le 98321 \end{cases} \tag{17}$$

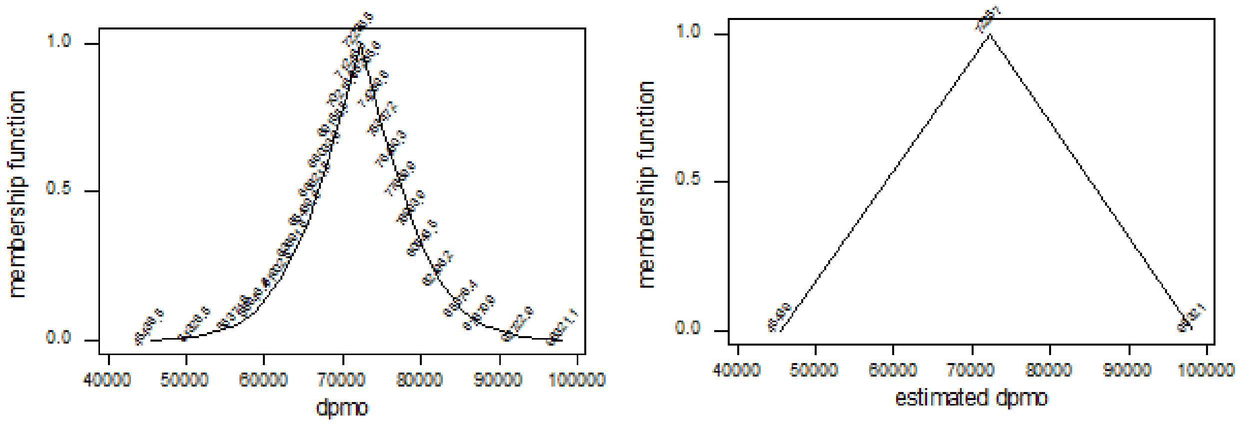

Fig. 4. DPMO and estimated DPMO

As you can see in Fig. 5 the vertex of fuzzy DPMO is at $x = d$ and the vertex of customer need is at $x = c$. A.D. represents the total area under the graph of fuzzy DPMO. A.R. is the area under the graph of fuzzy DPMO, but to the right of the vertical line through $x = c$. We choose a value of $\phi \in (0,1)$ and our decision rule is: if $\frac{A.R.}{A.D.} \ge \varphi$, then reject the process, otherwise do not reject that. Notice in this method we need two numbers β, the significant level of the test, and φ to judge the relative size of A_R. Let's in this paper choose $\varphi = 0.4$. Surely, $\varphi \geq 0.5$ is not acceptable. Notice that in Fig. 5 we get $\dfrac{A.R.}{A.D.} \geq 0.5$ when $x = d$ lies to the right of $x = c$.

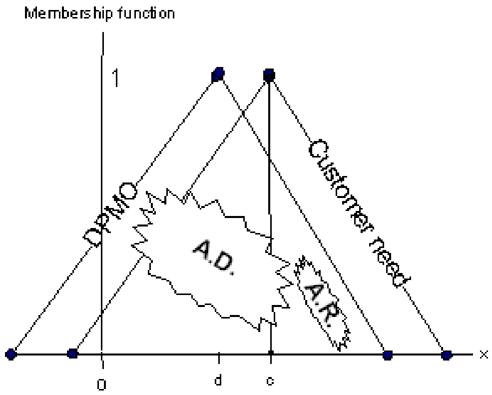

Fig. 5. Fuzzy estimated DPMO in comparison with fuzzy customer need in decision making

Example 3. Let n=1000 and all assumptions be as the same as Example 1. Consider customer need is a fuzzy number with shape and membership function like (Fig. 6) and (Eq.18) respectively.

$$\begin{cases} (x-50000)/(75000-50000) & , 50000 \leq x \leq 75000 \\ 1 & , x=75000 \\ (10000-x)/(10000-75000) & , 75000 \leq x \leq 10000 \end{cases} \qquad (18)$$

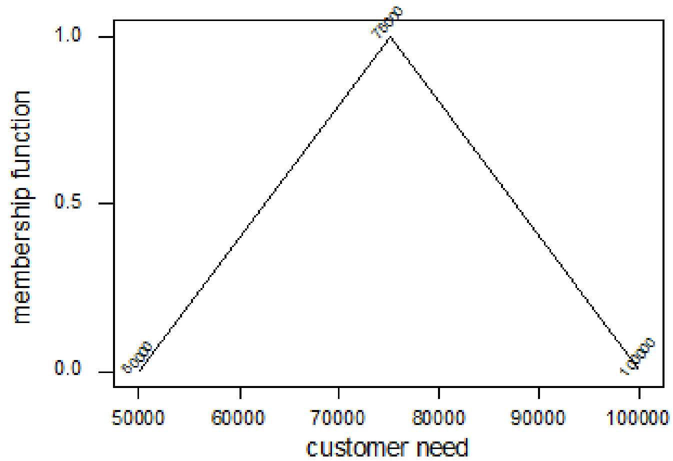

Fig. 6. Membership function of the fuzzy customer need

So Figure 5 in Example 3 is shown as Fig. 7.

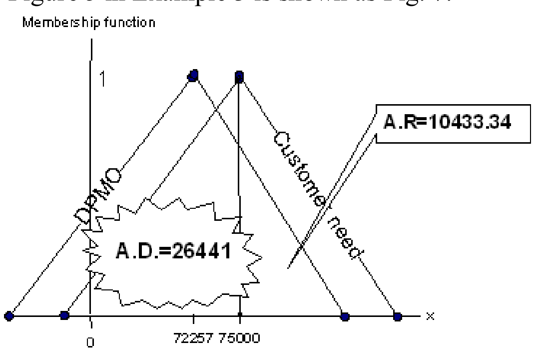

Fig.7. Illustrates Fig. 5 in Example 3

Now after doing some simple calculations decision rule leads to acceptance of the process as follows:

$$\frac{A.R.}{A.D.} = \frac{10433.34}{26441} = 0.394589 \leq 0.4$$

Since the result of the example is not rejected and also since the proposed estimation is pessimistic, the crisp DPMO must be smaller than crisp customer need. So, in exact decision making we should accept the process performance.

6. Conclusion

Since DPO, yield and DPMO are estimated using sample data, it is of interest to obtain confidence intervals rather than simple point estimates to calculate them. To find fuzzy estimates for DPO, yield and discrete DPMO, a new algorithm based on Buckley's approach named FESSM is introduced. The final results of the proposed algorithm not only contain point estimates but also interval estimates and hence provide us with more information. The diminishing impact of a larger sample size on amount and fuzziness of DPMO is also figured. A new method to compare fuzzy estimated DPMO with fuzzy customer need is introduced and numerical examples are given to illustrate the performance of the new algorithm and new comparing method. Future research may estimate continuous DPMO.

References

[1] Brefogle., Forrest W, Implementing Six Sigma, John Wiley & Sons,(1964), 0-471-26572-1.

[2] Buckley., J.J, Fuzzy Statistics: regression and prediction, Soft Computing 9 (2005) 764-775.

[3] Buckley., J.J, Fuzzy Statistics: hypothesis testing, Soft computing 9 (2005) 512-518.

[4] Buckley., J.J, Eslami., E, Uncertain probabilities I : the discrete case, Soft Computing 7 (2003) 500-505.

[5] Buckley., J.J, Eslami., E, Uncertain probabilities Π : the continuous case, Soft Computing 7 (2003) 500-505.

[6] Chen., K. S, Chen., T. W, (2008). Multi-process capability plot and fuzzy inference evaluation. International Journal of Production Economics, 111(1), 70–79.

[7] Chen., T. W, Chen., K. S, Lin., J. Y, (2003). Fuzzy evaluation of process capability for Bigger-the-Best type products. International Journal of Advanced Manufacturing Technology, 21, 820–826.

[8] Chen., T. W, Lin., J. Y, Chen., K. S, (2003). Selecting a supplier by fuzzy evaluation of capability indices C_{pm}. International Journal of Advanced Manufacturing Technology, 22, 534–540.

[9] Hsu., B. M, Shu., M. H, (2008). Fuzzy inference to assess manufacturing process capability with imprecise data. European Journal of Operational Research, 186(2), 652–670.

[10] Hsu., B. M, Wu., C. W, Shu., M. H, (2008). Generalized confidence intervals for the process capability index C_{pm}. Metrika, 68, 65–82.

[11] Hsu., Y. C, Pearn., W. L., Wu., P. C, (2008). Capability adjustment for gamma processes with mean shift consideration in implementing six-sigma program. European Journal of Operational Research, 191, 517–529.

[12] Kahraman., C, Kay., I, (2008). Fuzzy process capability indices for quality control of irrigation water. Stochastic Environmental Research and Risk Assessment. doi:10.1007/s00477-008-0232-8.

[13] Kahraman., C, Kay., I, (2009). Fuzzy process accuracy index to evaluate risk assessment of drought effects ın Turkey. Human and Ecological Risk Assessment: An International Journal, 15(4), 789–910.

[14] Kotz., S, Johnson., N, Process capability indices—a review 1992–2000, Journal of Quality Technology 34 (2002) 2–19.

[15] Kotz., S, Process Capability Indices, Chapman and Hall, New York, 1993.

[16] Montgomery., D.C, Introduction to Statistical Quality Control, John Wiley & Sons, New York, 2001.

[17] Pande., P. S, Neuman., R. P, Roland R. Cavanagh, The Six Sigma Way. New York, McGraw- Hill, (2000), p. xi.

[18] Parchami., A, Mashinchi., M, (2007). Fuzzy estimation for process capability indices. Information Sciences, 177, 452–1462.

[19] Parchami., A., Mashinchi., M, Maleki., H. R, (2006). Fuzzy confidence interval for fuzzy process capability index. Journal of Intelligent and Fuzzy Systems, 17, 287–295.

[20] Parchami., A, Mashinchi., M, Yavari., A. R, Maleki., H. R, (2005). Process capability indices as fuzzy numbers. Austrian Journal of Statistics, 34(4), 391–402.

[21] Wu., C. W, (2008). Assessing process capability based on Bayesian approach with subsamples. European Journal of Operational Research, 184(1), 207–228.

[22] Wu., C. W, (2009). Decision-making in testing process performance with fuzzy data. European Journal of Operational Research, 193(2), 499–509.

[23] Wu., C. W, Pearn., W. L, (2008). A variables sampling plan based on C_{pmk} for product acceptance determination. European Journal of Operational Research, 184(2), 549–560.