Measuring and Analyzing the Bullwhip Effect in a Two-Product and Two Echelon Supply Chain Using Control Theory Approach

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Abstract

Coordination is very important in supply chain management and it is one of the main factors in supply chain profitability. Bullwhip effect is one of the basic obstacles to achieve coordination in supply chains and reduction of this phenomenon has an important role in supply chain harmony. The other side, costs of supply chain can be mitigated and customer service level can be increased by reducing of bullwhip effect. Because measurement of bullwhip effect is very important in analysing and controlling of it, providing equations to investigate bullwhip effect behaviour based on real world supply chain conditions is necessary. The previous studies mostly concentrate on single product supply chain and few studies have been done on supply chains with more than one product. Here we quantify and investigate the bullwhip effect in a two-echelon supply chain with two products using control theory approach. Due to the relationship between demands of two products in our proposed supply chain, first order vector auto regressive model is used as demand process of the products. We also apply moving average method for lead-time demand forecasting within the "order up to" replenishment policy. We derive a closed form bullwhip measure and then bullwhip effect in a two-product supply chain is discussed and illustrated through a numerical example.

Keywords: Bullwhip Effect, Order-Up-To Policy, Supply Chain, System Engineering, Vector Auto Regressive Model.

1. Introduction

Today, outsourcing is so important due to its effect on the supply chain benefits. In fact, we encounter a number of suppliers, producers, and distributers, which work together with a suitable harmony to provide more gains for supply chain. Demand damping is a main barrier to achieve coordination within different stages of supply chain. Many companies have observed increasing fluctuation in orders while moving up from downstream sites (like distributers) to upstream sites (like suppliers). The result is a loss of supply chain profitability. Since the first study on bullwhip effect content by Forrester (1958) the bullwhip effect has been mentioned in a large number of researches. Sucky (2009) divided researches on the bullwhip effect into six general categories: (i) papers aiming at a quantification of the bullwhip effect, (ii) works focusing on analyzing and identifying the causes of the bullwhip effect, (iii) studies observing the bullwhip effect in some industries or in numerous examples from individual products and companies, (iv) papers addressing methods for reducing the bullwhip effect, (v) works focusing on simulating the system behaviour and (vi) papers focusing on experimental validation of the bullwhip effect.

Sterman (1989) developed Beer game as a piece of evidence for existence demand amplification in supply chains. Lee et al. (1997) introduced five main causes of this phenomenon: demand forecast updating, order batching, price fluctuation, rationing, and non-zero lead-time. Understanding these causes of the bullwhip effect can be useful for managers to find suitable solutions for haltering and controlling of its consequences.

Chen et al. (2000 a, b) quantified and derived a lower bound for the bullwhip effect in a simple supply chain for two cases of forecasting methods: moving average and exponential smoothing. Dejonckheere et al. (2003) proposed a control theory approach for measuring bullwhip effect and suggested a new general replenishment rule that can reduce variance amplification significantly. Disney and Towill (2003) introduced an ordering policy that results in taming bullwhip effect. Zhang (2004) considered three forecasting methods for a simple inventory control system and presented three measures for bullwhip effect based on three forecasting methods. Kim et al. (2006) investigated stochastic lead-time and investigated role of information sharing on bullwhip effect. Chandra and Grabis (2005) measured

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bullwhip effect when order size is calculated according to multiple step forecasts using autoregressive models. Luong (2007) investigated effects of autoregressive coefficient and lead-time on bullwhip effect when minimum expected mean squares of error forecasting method is used. Luong and Phien (2007) research was based on order of autoregressive demand pattern. They showed that in high order of demand pattern, bullwhip effect could be reduced when lead-time decreases. Makui and Madadi (2007) utilized the Lyapunov exponent and provided a measure for bullwhip effect. They presented useful results on the behavior of the bullwhip effect by investigating the mathematical relationships. Jaksic and Rusjan (2008) demonstrated that certain replenishment policies could be inducers of the bullwhip effect. Chaharsooghi and Sadeghi (2009) considered a two-product supply chain and quantified bullwhip effect measure using a statistical approach. They concluded that there is no explicit expression for the bullwhip effect measure, when statistical method is used for quantifying of the bullwhip effect. Consequently, bullwhip effect measure could be calculated for only limited cases. Wu et al. (2011) utilized agent-based model and simulation (ABMS), as one of the scientific and dynamic research methods for complex systems to establish a supply chain model and determine its abundant bullwhip effect phenomenon under swarm platform. Based on their analysis, it was proved that the ABMS is the effective way to study the bullwhip effect in complex supply chain. Nepal et al. (2012) presented an analysis of the bullwhip effect and net-stock amplification in a three-echelon supply chain considering step-changes in the production rates during a product's life-cycle demand. Using a simulation approach, the analysis was focused around highly complex and engineered products, which have relatively long production life cycles and require significant capital investment in manufacturing. Fazel Zarandi and Gamasaee (2013) investigated reducing of the bullwhip effect in fuzzy environments by means of type-2 fuzzy methodology. In order to reduce the bullwhip effect in a supply chain, they proposed a new method for demand forecasting. Zotteri (2013) analyzed the empirical demand data for fast moving consumer goods to measure the bullwhip effect. The data consisted of the sell-in from a large manufacturer to the retailers and the sell-out from a retailer to the consumers. The findings showed that the bullwhip-effect could be substantial. Due to the importance of forecasting method in bullwhip effect reduction, Jaipuria and Mahapatra (2014) proposed an integrated approach of Discrete Wavelet Transforms Analysis and Artificial Neural Network denoted as DWT-ANN for demand forecasting. Their model was tested and validated by conducting a comparative study between Autoregressive Integrated Moving Average (ARIMA) and proposed DWT-ANN model. The analysis indicates that the mean square error of DWT-ANN is comparatively less than that of the ARIMA model. Li et al. (2014) studied the damped trend forecasting method and its bullwhip generating behaviour when used within the Order-Up-To (OUT) replenishment policy.Using z-transform transfer functions they determine complete stability criteria for the damped trend forecasting method and showed that this forecasting mechanism is stable for a much larger proportion of the parametrical space than is generally acknowledged in the literature. Further, they demonstrated the damped trend OUT system sometimes will generate bullwhip and sometimes it will not. Buchmeister et al. (2014) simulated a simple three-stage supply chain using seasonal and deseasonalized time series of the market demand data in order to identify, illustrate and discuss the impacts of different level constraints on the bullwhip effect. The results were presented for different overall equipment effectiveness (OEE) and constrained inventory policies.

Although many researches have been done on the bullwhip effect, more investigations are still needed to study it, minimize its effect, and quantify it in order to provide solutions for complex supply chains. This research considers a two-echelon supply chain consisting of one retailer and one supplier or producer. This supply chain produces two products in which demand pattern is based on first order vector autoregressive (VAR(1)) model. Ordering policy for each product is "order up to policy" and forecasting method is "moving average". According to these assumptions, bullwhip effect is quantified in a two-product supply chain and a closed form for bullwhip effect calculations is presented using control theory approach. Then, it is described and analysed via a numerical example.

2. Supply Chain Assumptions

In this paper, a two-stage supply chain with two products is taken into consideration. Demand pattern is described by time series model, forecasting method is moving average and ordering policy is according to order up to policy. Detailed description of these propositions are explained in the next parts.

2.1. Notations

We use several parameters to quantify bullwhip effect measure as follows:

- D customer demand
- Q order quantity at the beginning of period t
- $Var(D)$ variance of demand
- $Var(Q_t)$ variance of order quantity
- $VAR(1)$ first order vector autoregressive
- $D_{x,t}$ demand of product x in period t
- $D_{y,t}$ demand of product y in period t
- S_t order-up-to level of period t
- $\phi_{..}$ autocorrelation coefficients

$\varepsilon_{x,t}$ forecast error of product x in period t

$\varepsilon_{y,t}$ forecast error of product y in period t

L order lead-time

$\hat{D}_t^L$ lead time demand forecast

$\hat{\sigma}_t^L$ standard deviation of lead time demand forecast

p number of periods in moving average forecast

2.2. Bullwhip effect measure

According to the previous researches on the bullwhip effect measure (like Chen et al. (2000 a, b) Kim et al. (2006) and Sucky (2009)) bullwhip ratio can be calculated by equation (1):

$$BE = \frac{Var(Q_t)}{Var(D)} \tag{1}$$

in which $Var(Q_t)$ is variance of retailer orders and $Var(D)$ is variance of the customer demand. Therefore, to provide bullwhip effect measures we must prepare equations for two mentioned terms.

2.3. The VAR(1) demand process with two products

We considered that there are two products in our supply chain with depended demand. Therefore we use a first order vector autoregressive (VAR(1)) model for representing of this relationship between demand of products. The VAR(1) time series model is given based on Wei (1990):

$$\begin{aligned} D_{x,t} &= \phi_{xx} D_{x,t-1} + \phi_{xy} D_{y,t-1} + \varepsilon_{x,t} \\ D_{y,t} &= \phi_{yx} D_{x,t-1} + \phi_{yy} D_{y,t-1} + \varepsilon_{y,t} \end{aligned} \tag{2}$$

Here we can see that demand for product x at time t, is given by the sum of three components. The first component i.e. $\phi_{xx} D_{x,t-1}$ is an autoregressive term of one period with itself. The second term i.e. $\phi_{xy} D_{y,t-1}$ is an autoregressive term with previous realisation of product y. The final term i.e. $\varepsilon_{x,t}$ is independently and identically distributed (white noise) random process. The demand process of the second product, product y, is simply a mirror image of the demand process for product x. $\varepsilon_{\{x,y\},t}$ can be regarded as the forecast error and we assume that is has a zero mean and is unit variance. We assume, from now on, that the error terms are uncorrelated as this simplifies the mathematics considerably. In order for the VAR(1) demand process to be stationary; the following criteria must be held:

$$\left| \frac{(\phi_{xx} + \phi_{yy}) \pm \sqrt{(\phi_{xx} - \phi_{yy})^2 + 4\phi_{xy}\phi_{yx}}}{2} \right| < 1 \tag{3}$$

Chaharsooghi and Sadeghi (2009) showed that in a VAR(1) demand process, variance of demand for each product in the stationary conditions is given by equation (4):

$$Var[D_i] = \frac{\varphi_{ii}\varphi_{jj}\left(\varphi_{ij}^2 - 1\right) - \varphi_{jj}^2\left(1 + \varphi_{ij}\varphi_{ji}\right) + \varphi_{ii}\varphi_{jj}^3 - \left(1 + \varphi_{ij}^2\right)\left(\varphi_{ij}\varphi_{ji} - 1\right)}{\left(\varphi_{ii}\varphi_{jj} - \varphi_{ij}\varphi_{ji} - 1\right)\left(\varphi_{ii} + \varphi_{ij}\varphi_{ji} + \varphi_{jj} - \varphi_{ii}\varphi_{jj} - 1\right)\left(1 + \varphi_{ii} - \varphi_{ij}\varphi_{ji} + \varphi_{jj} + \varphi_{ii}\varphi_{jj}\right)} \tag{4}$$

where, for product x, $i=x$ and $j=y$; for product y, $i=y$ and $j=x$. Therefore we can use equation (4) as denominator of equation (1) in bullwhip effect measurement.

2.4. The OUT policy with moving average forecasting

We consider a single echelon of a supply chain which uses the Order-Up-To (OUT) policy to generate replenishment orders to maintain inventory levels. The forecasting method is also moving average. The goal of order up to policy is to bring the actual inventory towards the desired inventory. The order quantity that retailer places to the supplier is given by:

$$Q_t = S_t - S_{t-1} + D_t \tag{5}$$

Using base stock policy, order up to level, at the beginning of period t can be determined by equation (6):

$$S_t = \hat{D}_t^L + z.\hat{\sigma}_t^L \tag{6}$$

In equation (6), z is standard normal score and can be determined by normal table based on the desired service level required from the inventory system. Replacing equation (6) in equation (5) concludes equation (7) that is order quantity in period t:

$$Q_t = \hat{D}_t^L - \hat{D}_{t-1}^L + z(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L) + D_t \tag{7}$$

based on moving average forecasting method, $\hat{D}_t^L$ can be calculated by equation (8):

$$\hat{D}_t^L = L\left(\frac{\sum_{j=0}^{p-1} D_{t-j}}{p}\right) \tag{8}$$

As each of products is ordered independently, equation (8) can be applied to both of products with a suitable change in their parameters. Figure 1 highlights a block diagram of the two product supply chain we have just described.

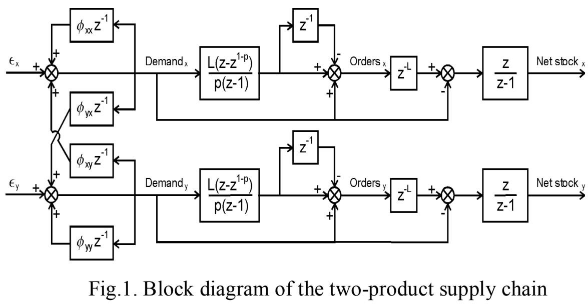

Fig.1. Block diagram of the two-product supply chain

Figure 1 shows that our system is a multi-input, multi-output system, so we need multiple transfer functions to describe its dynamics. As we have assumed the white noise processes ε_x and ε_y are uncorrelated, there is no interaction between them, thus their individual combinations to the variance expressions can simply be added together. Based on Disney and Towill (2003) we can provide equations for quantifying variance of orders as follows. Let's consider the combination of ε_i to $Orders_i$. The transfer function is:

$$\frac{Orders_i(z)}{\varepsilon_i(z)}=\frac{z^{1-p}\left((L+P)z^p-L\right)\left(z-\phi_{jj}\right)}{p\left(z^2-\phi_{ij}\phi_{ji}+\phi_{ii}\phi_{jj}-z\left(\phi_{ii}+\phi_{jj}\right)\right)} \tag{9}$$

This has the following time domain impulse response:

$$o_{t,\varepsilon_i}=\frac{2^{-1-n}\Gamma^{-p}\xi^{-p}}{p\Gamma}\begin{pmatrix}(L+p)\gamma^p\xi^p\left(\beta\xi^n+\gamma^n\eta\right)+\\ 2^pL\begin{pmatrix}2\left(\gamma^p\varsigma^n-\gamma^n\xi^p\right)\varphi_{jj}H[n-p]+\\ \left(\gamma^{1+n}\xi^p-\gamma^p\xi^{1+n}\right)H[1+n-p]\end{pmatrix}\end{pmatrix} \tag{10}$$

where the following substitutions have been made as the equation is rather lengthy:

$\Gamma=\sqrt{\phi_{ii}^2+4\phi_{ij}\phi_{ji}-2\phi_{ii}\phi_{jj}+\phi_{jj}^2}$, $\beta=\Gamma+\phi_{ii}-\phi_{jj}$, $\chi=\phi_{ii}+\phi_{jj}-\Gamma$, $\xi=\Gamma+\phi_{ii}+\phi_{jj}$, $\eta=\Gamma-\phi_{ii}+\phi_{jj}$ and $H[w]$ is the Heaviside step function (that is, $H[w]=0$ if $w<0$, 1 otherwise).

Equation (10) was obtained by taking the inverse z-transform of equation (9). The contribution of ε_i to the variance of orders for product i is given by the sum of the squared impulse response (Disney and Towill (2003)). Because of the Heaviside step-function properties, it is useful to split the sum into the following two parts:

$$\sigma_{i,\varepsilon_i}^2=\begin{pmatrix}\sum_{n=0}^{p-1}\left(\frac{2^{-1-n}(L+p)\left(\eta(\gamma)^n+(\beta)(\xi)^n\right)}{p\Gamma}\right)^2+\\ \sum_{n=p}^{\infty}\left(\frac{2^{-1-n}\gamma^{-p}\xi^{-p}\begin{pmatrix}\beta\gamma^p\xi^n\left((L+p)\xi^p-2^pL\right)+\\ \gamma^n\left((L+p)\gamma^p-2^pL\right)\xi^p\eta\end{pmatrix}}{p\Gamma}\right)^2\end{pmatrix} \tag{11}$$

Equation (11) has the following closed form represented by equation (12):

$$\sigma_{i,\varepsilon_i}^2=\frac{4^{-p}\begin{pmatrix}\frac{2(L+p)^2\beta\left(-4^p+(\gamma\xi)^p\right)\eta}{\gamma\xi-4}-\\ \frac{(L+p)^2\beta^2\left(4^p-\left(\xi^2\right)^p\right)}{\xi^2-4}-\\ \frac{4^p(L+p)^2\eta^2}{\gamma^2-4}+\frac{(L+p)^2\left(\gamma^2\right)^p\eta^2}{\gamma^2-4}+\\ p^2\left(\frac{\beta^2\xi^{2p}}{4-\xi^2}-\frac{2\beta\gamma^p\xi^p\eta}{\gamma\xi-4}-\frac{\gamma^{2p}\eta^2}{\gamma^2-4}\right)+\\ 2Lp\begin{pmatrix}\frac{\beta^2\xi^p\left(2^p-\xi^p\right)}{\xi^2-4}+\frac{\beta\left(2^p\xi^p+\gamma^p\left(2^p-2\xi^p\right)\right)\eta}{\gamma\xi-4}+\\ \frac{\gamma^p\left(2^p-\gamma^p\right)\eta^2}{\gamma^2-4}\end{pmatrix}+\\ L^2\begin{pmatrix}\frac{2\beta\left(2^p-\gamma^p\right)\left(\xi^p-2^p\right)\eta}{\gamma\xi-4}-\\ \frac{\beta^2\left(4^p-2^{1+p}\xi^p+\xi^{2p}\right)}{\xi^2-4}-\frac{\left(4^p-2^{1+p}\gamma^p+\gamma^{2p}\right)\eta^2}{\gamma^2-4}\end{pmatrix}\end{pmatrix}}{p^2\Gamma^2} \tag{12}$$

Now we consider the influence of ε_j on $Orders_i$. The transfer function is:

$$\frac{Orders_i(z)}{\varepsilon_j(z)}=\frac{z\left(L+p-Lz^{-p}\right)\phi_{ij}}{p\left(z^2-\phi_{ij}\phi_{ji}+\phi_{ii}\phi_{jj}-z\left(\phi_{ii}+\phi_{jj}\right)\right)} \tag{13}$$

Taking the inverse z-transform yields the following time domain impulse response:

$$o_{t,\varepsilon_j}=\frac{2^{-n}\gamma^{-p}\xi^{-p}\varphi_{ij}\begin{pmatrix}(L+p)\gamma^p\xi^p\left(\xi^n-\gamma^n\right)+\\ 2^pL\left(\gamma^n\xi^p-\gamma^p\xi^n\right)H[n-p]\end{pmatrix}}{p\Gamma} \tag{14}$$

The contribution of ε_j to the variance of $Orders_i$ is given by the sum of the squared impulse response (Disney and Towill (2003)). That is:

$$\sigma_{i,\varepsilon_j}^2=\sum_{n=0}^{p-1}\left(\frac{2^{-n}(L+p)\left(\xi^n-\gamma^n\right)\varphi_{ij}}{p\Gamma}\right)^2+\sum_{n=p}^{\infty}\left(\frac{2^{-n}\varphi_{ij}\left(2^pL\left(\gamma^{n-p}-\xi^{n-p}\right)-(L+p)\left(\gamma^n-\xi^n\right)\right)}{p\Gamma}\right)^2 \tag{15}$$

which has the closed form represented by equation (16). A more compact and explicit form of equation could not be provided due to the complexity of the relationships.

The variance of the orders for product i is given by $Var[Orders_i]=\sigma_{i,\varepsilon_i}^2+\sigma_{i,\varepsilon_j}^2$. However, the following alternative formulation represented by equation (17) has been concluded in a more summary format:

$$\sigma_{i,\varepsilon_j}^2=\frac{4^{1-p}\phi_{ij}^2\begin{pmatrix}p^2\left(2\gamma^p\left(\gamma^2-4\right)\xi^p\left(\xi^2-4\right)-\left(\gamma^2-4\right)\xi^{2p}(\gamma\xi-4)-\gamma^{2p}(\gamma\xi-4)\left(\xi^2-4\right)\right)-\\ L^2\begin{pmatrix}4^{1+p}\gamma^2+16\gamma^{2p}-2^{3+p}\gamma^{2+p}-2^{3+2p}\gamma\xi+4^p\gamma^3\xi+2^{3+p}\gamma^{1+p}\xi-4\gamma^{1+2p}\xi+\\ 4^{1+p}\xi^2-2^{1+2p}\gamma^2\xi^2-4\gamma^{2p}\xi^2+2^{1+p}\gamma^{2+p}\xi^2+4^p\gamma\xi^3-2^{1+p}\gamma^{1+p}\xi^3+\gamma^{1+2p}\xi^3+\\ \left(\gamma^2-4\right)\xi^{2p}(\gamma\xi-4)-2\left(\gamma^2-4\right)\xi^p\left(2^p(\gamma-\xi)\xi+\gamma^p\left(\xi^2-4\right)\right)\end{pmatrix}+\\ 2Lp\left(\left(\gamma^2-4\right)\xi^p\left(2^p(\gamma-\xi)\xi+2\gamma^p\left(\xi^2-4\right)\right)-\left(+\gamma^2-4\right)\xi^{2p}(\gamma\xi-4)-\gamma^p\left(\xi^2-4\right)\left(2^p\gamma(\gamma-\xi)+\gamma^p(\gamma\xi-4)\right)\right)+\\ (L+p)^2\begin{pmatrix}\gamma\xi\left(2^{3+2p}-4\left(\gamma^2\right)^p+\left(\left(\gamma^2\right)^p-4^p\right)\xi^2-4\left(\xi^2\right)^p\right)-4\xi^2\left(4^p+\left(\gamma^2\right)^p-2(\gamma\xi)^p\right)-\\ \gamma^3\xi\left(4^p-\left(\xi^2\right)^p\right)+16\left(\left(\gamma^2\right)^p-2(\gamma\xi)^p+\left(\xi^2\right)^p\right)-2\gamma^2\left(\xi^2\left((\gamma\xi)^p-4^p\right)+2\left(4^p-2(\gamma\xi)^p+\left(\xi^2\right)^p\right)\right)\end{pmatrix}\end{pmatrix}}{p^2\left(\gamma^2-4\right)\Gamma^2(\gamma\xi-4)\left(\xi^2-4\right)} \tag{16}$$

$$Var\left[Orders_i\right]=\left(\begin{array}{l}\sum_{n=0}^{p-1}\left(\dfrac{2^{-2(1+n)}\left(L+p\right)^2\left(\left(\beta\xi^n+\chi^n\eta\right)^2+4\left(\chi^n-\xi^n\right)^2\varphi_{ij}^2\right)}{p^2\Gamma^2}\right)+\\ \sum_{n=p}^{\infty}\left(\dfrac{2^{-2(1+n)}}{p^2\Gamma^2}\left(\begin{array}{l}\chi^{-2p}\xi^{-2p}\left(\beta\chi^p\xi^n\left(-2^pL+\left(L+p\right)\xi^p\right)+\chi^n\left(-2^pL+\left(L+p\right)\chi^p\right)\xi^p\eta\right)^2+\\ 4\left(\left(L+p\right)\left(\chi^n-\xi^n\right)+2^pL\left(-\chi^{n-p}+\xi^{n-p}\right)\right)^2\varphi_{ij}^2\end{array}\right)\right)\end{array}\right) \tag{17}$$

This closed form is valid across all lead-times *L* and moving average constant, *p*.

$$Var[Orders_i]=\frac{1}{(p\Gamma)^2}\left(\begin{array}{l} L^2\left(\begin{array}{l}\dfrac{\beta^2\left(1+2^{-p}\xi^{-2p}\left(\xi^2\right)^p\left(2^p-2\xi^p\right)\right)}{4-\xi^2}+\dfrac{2^{1-2p}\beta\left(-2^{1+2p}+2^p\xi^p+\left(\chi\xi\right)^p+\chi^p\left(2^p-\xi^p\right)\right)\eta}{\chi\xi-4}-\dfrac{\eta^2}{\chi^2-4}-\dfrac{4\phi_{ij}^2}{\chi^2-4}+\dfrac{16\phi_{ij}^2}{\xi\chi-4}-\dfrac{4\phi_{ij}^2}{\xi^2-4}\\ -\dfrac{2^{2-p}\xi^{-2p}\left(\xi^2\right)^p\left(2^p-2\xi^p\right)\phi_{ij}^2}{\xi^2-4}+\dfrac{2^{3-2p}\chi^p\left(\xi^p-2^p\right)\phi_{ij}^2}{\chi\xi-4}-\dfrac{2^{3-2p}\left(2^p\xi^p+\left(\chi\xi\right)^p\right)\phi_{ij}^2}{\chi\xi-4}-\dfrac{2^{-p}\chi^{-2p}\left(\chi^2\right)^p\left(2^p-2\chi^p\right)\left(\eta^2+4\phi_{ij}^2\right)}{\chi^2-4}\end{array}\right)+\\ p^2\left(\dfrac{\beta^2}{4-\xi^2}-\dfrac{2^{1-2p}\beta\eta\left(4^p+\chi^p\xi^p-\left(\chi\xi\right)^p\right)}{\chi\xi-4}-\dfrac{\eta^2}{\chi^2-4}+4\phi_{ij}^2\left(\dfrac{1}{4-\chi^2}+\dfrac{2^{1-2p}\left(\chi\xi\right)^p}{4-\chi\xi}-\dfrac{2}{4-\chi\xi}-\dfrac{2^{1-2p}\chi^p\xi^p}{4-\chi\xi}+\dfrac{1}{4-\xi^2}\right)\right)+\\ 2Lp\left(\begin{array}{l}\dfrac{\beta^2\left(1-2^{-p}\xi^{-p}\left(\xi^2\right)^p\right)}{4-\xi^2}+\dfrac{4^{-p}\beta\left(2^p\xi^p+\chi^p\left(2^p-2\xi^p\right)-2\left(4^p-\left(\chi\xi\right)^p\right)\right)\eta}{\chi\xi-4}+\dfrac{\left(1-2^{-p}\chi^{-p}\left(\chi^2\right)^p\right)\eta^2}{4-\chi^2}+\\ 4\phi_{ij}^2\left(\dfrac{1}{4-\chi^2}+\dfrac{2}{\chi\xi-4}+\dfrac{1}{4-\xi^2}+4^{-p}\left(\dfrac{2^p\chi^{-p}\left(\chi^2\right)^p}{\chi^2-4}+\dfrac{2^p\xi^{-p}\left(\xi^2\right)^p}{\xi^2-4}-\dfrac{\chi^p\left(2^p-2\xi^p\right)}{\chi\xi-4}+\dfrac{2^p\xi^p+2\left(\chi\xi\right)^p}{4-\chi\xi}\right)\right)\end{array}\right)\end{array}\right) \tag{18}$$

Equation (18) further simplifies to the following:

$$Var[Orders_i]=\frac{\left(\begin{array}{l}\dfrac{\left(\beta^2+4\phi_{ij}^2\right)\left(2L^2\left(1-\left(\dfrac{\xi}{2}\right)^p\right)+2Lp\left(1-\left(\dfrac{\xi}{2}\right)^p\right)+p^2\right)}{4-\xi^2}+\dfrac{\left(\eta^2+4\phi_{ij}^2\right)\left(2L^2\left(1-\left(\dfrac{\chi}{2}\right)^p\right)+2Lp\left(1-\left(\dfrac{\chi}{2}\right)^p\right)+p^2\right)}{4-\chi^2}+\\ \dfrac{\left(\beta\eta-4\phi_{ij}^2\right)\left(2L^2\left(\left(\dfrac{\xi}{2}\right)^p+\left(\dfrac{\chi}{2}\right)^{pp}-2\right)+2Lp\left(\left(\dfrac{\xi}{2}\right)^p+\left(\dfrac{\chi}{2}\right)^p-2\right)-2p^2\right)}{\chi\xi-4}\end{array}\right)}{(p\Gamma)^2} \tag{19}$$

3. Quantifying the Bullwhip Effect

Consider equation (1) again that defines bullwhip effect relationship. As mentioned before, we can use equation (4) as variance of market demand, needed in denominator of equation (1). Moreover, equation (19) represents variance of orders and can be used as numerator of equation (1). Therefore bullwhip effect values can be measured by division of their results. In the next part, we complete Tables 1 and 2 in this manner. Now we want to prove a proposition in which expressed conditions that bullwhip effect can be removed from the proposed supply chain.

Proposition: When *p* approaches infinity, variance of orders approaches variance of demand. That is, when $p=\infty$ then $Var[Orders_i]=Var[D_i]$. Thus, when forecasts become equal to the long-term average demand, the bullwhip ratio is equal to one.

Proof: When *p* approaches infinity, we can derive equation (20) based on equation (19):

$$\lim_{p\to\infty}\left\{Var\left[Orders_i\right]\right\}=\frac{\left(\begin{array}{l}(\chi\xi-4)[(\beta\chi+\xi\eta)^2-4(\beta+\eta)^2]-\\ 4\left(\chi-\xi\right)^2[2\beta\eta-\left(4+\chi\xi\right)\varphi_{ij}^2]\end{array}\right)}{\left(4-\chi^2\right)\Gamma^2\left(\chi\xi-4\right)\left(\xi^2-4\right)} \tag{20}$$

Substituting β,η,χ,ξ we have:

$$\lim_{p\to\infty}\left\{Var\left[Orders_i\right]\right\}=\frac{\begin{array}{c}\varphi_{ii}\varphi_{jj}\left(\varphi_{ij}^2-1\right)-\varphi_{jj}^2\left(1+\varphi_{ij}\varphi_{ji}\right)+\\ \varphi_{ii}\varphi_{jj}^3-\left(1+\varphi_{ij}^2\right)\left(\varphi_{ij}\varphi_{ji}-1\right)\end{array}}{\begin{array}{c}\left(\varphi_{ii}\varphi_{jj}-\varphi_{ij}\varphi_{ji}-1\right)\left(\varphi_{ii}+\varphi_{ij}\varphi_{ji}+\varphi_{jj}-\varphi_{ii}\varphi_{jj}-1\right)\\ \left(1+\varphi_{ii}-\varphi_{ij}\varphi_{ji}+\varphi_{jj}+\varphi_{ii}\varphi_{jj}\right)\end{array}} \tag{21}$$

It is clear that equation (21) is equal to $Var[D_i]$ derived before by equation (4) .Therefore when $p\to\infty$ the bullwhip effect approaches one. That is: $Bullwhip_i=\dfrac{Var[Orders_i]}{Var[D_i]}\to 1$ and when $p=\infty$ then $Bullwhip_i=1$. So there is no bullwhip effect for product *i* in the supply chain if retailer uses as large as number of periods (*p*) in average calculations.

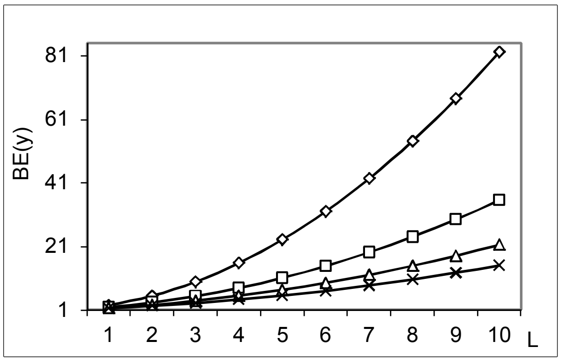

Fig. 2(b). Bullwhip effect variation with respect to *L* for product *y*

Figure 3(a) shows bullwhip effect measures of the product x with respect to the number of observations in lead-time demand forecasting, *p*. The upper curve is bullwhip ratio for L=4 and the others are for L=3, L=2, L=1, respectively. It is clear that more the observation results, the less the bullwhip effect. In addition, a sharp fall can be seen when the number of observations changes from 1 to 2. Moreover, Figure 3(a) shows that slope of the bullwhip effect curve decreases dramatically when lead-time increases. In other words if lead-time is large, then increasing *p* from 1 to 2 can reduce bullwhip effect considerably.

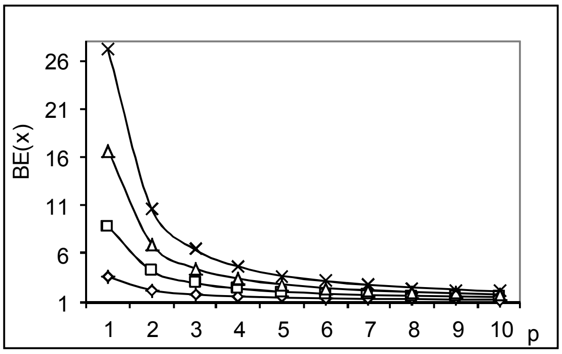

Fig. 3(a). Bullwhip effect variation with respect to *p* for product *x*

Figure 3(b) shows bullwhip effect curve for the product y with respect to number of observations in lead-time demand forecasting, *p*. It is clear that bullwhip effect decreases when number of observations increases. A dramatic fall can be seen when lead-time of the second product increases from 1 to 2. Slope of curves between bullwhip effects of two products differs in Figure 3(a) and Figure 3(b).

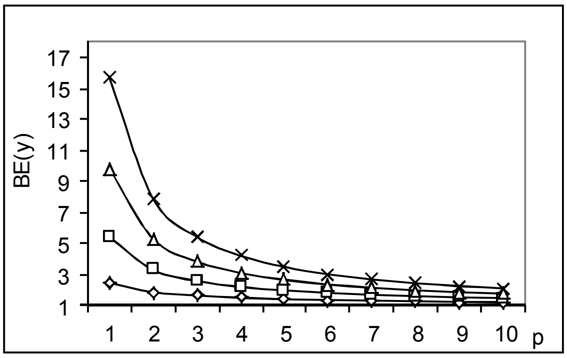

Fig. 3(b). Bullwhip effect variation with respect to *p* for product *y*

Finally, Figures 4(a) and 4(b) show bullwhip ratios for two products; separately for each product *x* and product *y* when *p* (y dimension of the curve) and *L* (x dimension of the curve) vary simultaneously. Also z dimension of curve shows bullwhip effect measure in a proposed value of *p* and *L*. In fact, when a retailer increases number of periods in his moving average calculations and supplier reduces lead-time of procurement, bullwhip effect decreases. So we have minimum of bullwhip measure in maximum value of *p* and minimum value of *L*. While *p* decreases or *L* increases, bullwhip effect measure increases, as we can see in Figures 4(a) and 4(b), the maximum point of the flat curve occurs in maximum measure of lead-time and the minimum measurer of moving average parameter (*p*). In fact coordination retailer and supplier in selection of *L* and *p* can reduce bullwhip effect of products.

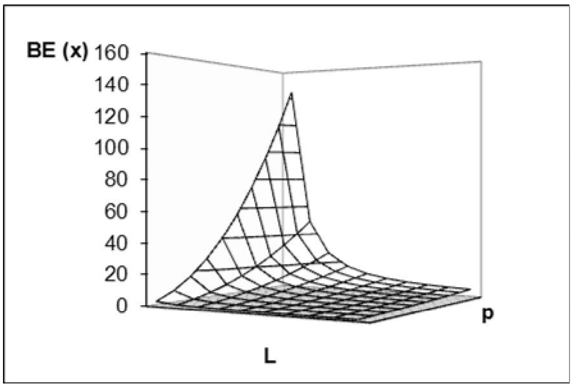

Fig. 4(a). Bullwhip effect variation with respect to *p* and *L* for product *x*

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