scenario approach to facility location. He suggests selecting facility locations to minimize expected cost;however he does not discuss the subject at length.

In any stochastic programming problem, one must determine which decision variables are first-stage and which are second-stage; that is, which variables must be set now and which may be set after the uncertainty has been resolved. In stochastic location modeling, locations are generally first-stage decisions while assignments of customers to facilities are second-stage decisions (Ghezvati, Jabal-Ameli, &Makui (2008) and Snyder, 2006). If both decisions occur in the first stage, most stochastic location problems can be reduced easily to deterministic problems.

Mirchandani, Oudjit, and Wong (1985) and Weaver and Church (1983) presented algorithms for a multi-scenario for P-median problem (PMP). Their algorithms effectively treat the problem as a deterministic PMP with |I|×|S| customers instead of |I|, where I is the set of customers and S is the set of scenarios. Louveaux (1986) introduced stochastic versions of the capacitated PMP and Capacitated facility location problem (CFLP) in which demand, production costs, and selling prices are arbitrary. Vidal and Goetschalckx (2000) discussed the importance of incorporating various types of uncertainty into global supply chain design decisions.

More formal stochastic programming techniques are used by Alonso-Ayuso, Escudero, Garı´n, Ortun˜o, and Pe´rez (2003), to solve multi-echelon supply chain design problems.

Kang and Kim (2010) considered an integrating inventory replenishment model and delivery planning in a two-level supply chain consisting of a supplier and a retailer. Shen and Qi (2007) proposed a single-product, single-period LRI problem with an approximate routing cost and solved the LRI model by a Lagrangian relaxation based solution algorithm, their model was introduced as a modified inventory-location model given in Daskin, Coullard, and Shen (2002). Chanchan, Zujun, and Huajun (2008) formulated a dynamic LRI problem in a closed loop supply chain solved by a two-phase heuristic algorithm. Ahmadi-Javid and Azad (2010) developed the model presented by Shen and Qi (2007). Their model simultaneously optimizes location, inventory and routing decisions without approximation, and is solved by a heuristic method based on a hybridization of tabu search and simulated annealing.

Moin, Salhi, and Aziz (2011) addressed an inventory routing, many-to-many distribution network consisting of an assembly plant and many distinct suppliers where each supplies a distinct product. Hiassat and Diabat (2011) studied the LRI problem with perishable product, through a multi-period model.

In this paper, we define a notation of a supply chain system considering stochastic demand and inert-depot transportation. Also some restrictions such as coverage radius and inventory capacity are assumed in the thinking process.

The structure of this paper is as follows. In Section 2, we present the formulation of the problem. Section 3 proposes solution procedure regarded to Benders' decomposition method. We present computational results and sensitivity analysis in Section 4. In Section 5, we summarize our conclusions and discuss avenues for future research.

2. Model Formulation

Let N be the set of customers, which face uniform distributed demands that are independent among customers. Let M be the set of potential sites for distribution centers. The firm pays a fixed location cost for opening a DC, as well as a holding cost for inventory. In this model since customers’ demand is stochastic, service level constraint is considered for that an arbitrary request arriving at a DC, should only have a (pre-specified) small probability of being lost.

The notice is that all distribution centers cannot service all of the customers due to considering special coverage restriction for each DC.Thus,if a customer cannot fall in the coverage radius,so the DC cannot service that customer. In this model,this point is considered which makes the model to be more realistic.

The other assumption applied in this study is inter-depot transportation. This point can enable DCs to have transportation products among each other. Due to stochastic demand which leads to inventory fluctuation in facilities, inter –depot transportation has risk-pooling effect on the supply chain network. Thus, holding inventory and shortage in the whole chain will be reduced. The sample of representation of a distribution network considering inter – depot transportation is illustrated in Figure 1.

In the previous location - inventory models the required order, for each DC’s, is the sum of two parts. First part is the sum of mean demand, assigned to the DC and the second part is the safety stock. A new approach for this model in this article can be proposed without dividing the order to two parts. By combining the two parts, let MI_j be the amount of the order DC j. In this section, we aim to develop a chance constraint programming in order to prevent demand shortage for each customer that is assigned to a DC. As it was defined earlier, the demand for each customer follows uniform distribution within the interval (a_i, b_i). For this purpose, anamount of product that a DC orders to cover each customer's demand (Q_{ij}) must satisfy the following constraint. By this way, the probability that each customer faces to demand shortage is at most a%.

2.1 Parameters and Decision Variables

Parameters

N index set of customer zones.

M index set of potential distribution sites .

L index set of potential plant site.

C_{ij} Cost of supplying one unit of demand to customer zone i from DC at site j.

$\overline{C}_{jk}$ Cost of supplying one unit of demand to DC at site j from plant k.

F_j Fixed cost for opening DC at site j.

G_k Fixed cost for opening plant at site k.

b_j Capacity for the potential DC at site j.

e_k Capacity for the potential plant at site k.

$U_i \sim (a_i, b_i)$ Uniform distribution function for demand of customer i.

λ_i Mean of demand per unit time for customer i. ($\lambda_i = \dfrac{a_i + b_i}{2}$)

h_i Holding cost per unit in DC at site j.

$$Z_{ij} = \begin{cases} 1 & \text{if the DC at site } j \text{ covers the customer site } i. \\ 0 & \textit{Otherwise} \end{cases}$$

Decision variables:

$$X_{ij} = \begin{cases} 1 & \text{if demand of customer } i \text{ satisfied by DC at site } j. \\ 0 & \text{Otherwise} \end{cases}$$

Q_{ii} : Amount of product that DC j orders to satisfy demand of customer i with the probability at least $1 - \alpha$.

Y_{jk} Amount of product that is transported from plant k to DC j.

$$U_j = \begin{cases} 1 & \text{if a DC is located at site } j. \\ 0 & \text{Otherwise} \end{cases}$$

$$V_k = \begin{cases} 1 & \text{if a plant is located at site } k. \\ 0 & \text{Otherwise} \end{cases}$$

$IDT_{j,j'}$ Total product shipped from DC j to DC j'.

2.2 Proposed Model

$$MinZ = \sum_{j\in M}\sum_{k\in L}\overline{C_{jk}} \times Y_{jk} + \sum_{j\in M}\sum_{i\in N}\lambda_i \times X_{ij} \times C_{ij} + \sum_{j\in M} F_j \times U_j + \sum_{k\in L} G_k \times V_k + \sum_{i\in N}\sum_{j\in M}\lambda_i \times X_{ij} \times LI \times \eta_{ij} \quad (5)$$

St:

$$\sum_{j\in M} X_{ij} = 1 \qquad \forall i \in N \quad (6)$$

$$X_{ij} \le Z_{ij} \times U_j \qquad \forall i \in N, \forall j \in M \quad (7)$$

$$\sum_{k\in L} Y_{jk} \le U_j \times b_j \qquad \forall j \in M \quad (8)$$

in sub-problem. If sub-problem becomes infeasible, feasibility cut will be produced in the master problem.
We adopt a solution procedure as Benders' Decomposition algorithm. In the proposed algorithm, master problem and sub-problem are formulated in regard to following indices and parameters.
Indices of algorithm:

$iter \in I$ Iterations of the algorithm

$p \subseteq I$ Set of iterations that sub-problem is feasible

$f \subseteq I$ Set of iterations that sub-problem is infeasible

Parameters of algorithm:

W_j Set of binary variables

$\text{fix\_W}_{j(iter)}$ Fixed value for W_j at iteration iter

$u_{j(iter)}$ Dual value in relation to constraints (8) and (9) at iteration iter

objsub_{iter} Value of objective function for sub-problem at iteration iter

LB Lower bound of problem

UB Upper bound of problem

Sub-problem:

Max

$$u_1 \times (\sum_{k \in L} Y_{jk} - fix\_U_j \times b_j) +$$

$$u_2 \times ([(1-\alpha)\times(b_i - a_i) + a_i] \times fix\_X_{ij} - Q_{ij})$$

Subject to all constraints expect those ones contain binary variables.

$$\sum_{k \in L} Y_{jk} \le fix\_U_j \times b_j \qquad \forall j \in M$$

$$Q_{ij} \ge [(1-\alpha)\times(b_i - a_i) + a_i] \times fix\_X_{ij}$$

Master problem:

Min λ

$$\lambda \ge Z_{\text{where bibary variables are fixed}} + u_1 \times (\sum_{k \in L} Y_{jk} - fix\_U_j \times b_j) +$$

$$u_2 \times ([(1-\alpha)\times(b_i - a_i) + a_i] \times fix\_X_{ij} - Q_{ij})$$

$$\sum_{j:fix\_W_j=0} W_j + \sum_{j:fix\_W_j=1} (1 - W_j) \ge 1 \qquad \forall f$$

In the objective function of master problem, α is a free variable that exists in optimality cuts. At first, master problem without α and cuts solves then optimal values are fixed in sub-problem. In turn, if the sub-problem is feasible, optimality cut will be placed in master problem or if it is infeasible, feasibility cut will be located in master problem. Consequently, master problem obtained by optimality cuts or feasibility cuts solves then optimal binary values are fixed again. This procedure continues until upper bound is equal to lower bound. Final solution is a global solution for original problem. For very big dimensions, stopping criteria may be used to avoid extreme run time, such as maximum iteration and minimum thegap between lower and upper bounds. We present a pseudo-code for the proposed procedure as follows:

1. $iter = 0$ *and* $p, f = \varnothing$, $LB = -\infty$, $UB = \infty$
2. *solve master problem*
3. $iter = iter + 1$
4. $fixW_{j(iter)} = W_j^*$
5. *solve subproblem*
6. *if infeasible then* $f = f \cup \{iter\}$
7. *else if* $p = p \cup \{iter\}$ $u_{j(iter)} =$ *dual constraints* (18) *and* (19)
8. $LB = \max\left\{LB, \text{obj}_{iter} - \sum_j Lo\,cost_j .fixW_j\right\}$
9. *solve master problem*
10. $UB = objmaster$
11. *if* $UB = LB$ *then* $obj^* = objmaster^*$
12. *else if go to step* 3

4. Computational Results

4.1 Benders' validation

We formulate the problem, and implement the proposed algorithm in GAMS 23.5 in relation with CPLEX® Solver version 12.2. In the first section, validity of GAMS codes are controlled via some test problems. All examples are generated randomly and they are solved by both regular GAMS and also Benders' decomposition method. From Table 1,we can see that there is no difference between all the objective functions. Therefore, it is obvious that there is no gap when different problems are solved. This implies that the proposed Benders' decomposition method as well as the regular GAMS solver is effective in solving the presented model.

(18)

(19)

(20)

Table 1
Validation of the proposed Benders' decomposition

|M|	|N|	|L|	No. of open DCs	No. of open plants	Total cost by Benders'	Total cost by GAMS
8	5	3	2	1	15221	15221
9	5	3	2	1	16044	16044
10	6	3	2	1	16215	16215
11	6	3	3	1	16899	16899
12	6	3	3	1	17451	17451

4.2 Sensitivity analysis

In this section, problems sets were generated randomly but systematically to capture a wide range of problems structures. The numbers of customer zones, potential DC and potential plants vary from 40 to 70, 6 to 12 and from

5. Conclusion and Future Directions

In this paper, we introduced notation of distribution network in a supply chain system while we have service level constraint and customer's demand is stochastic with Poisson distribution function. In the proposed model, we assumed that DCs have coverage radius restriction and also, inert-depot transportation. We formulated the problem as a nonlinear integer program. The advantages of the proposed model are as follows: considering coverage radius which yields more flexibility for the model, considering service level constraint that byusing it the required inventory can be computed automatically, it prevents inventory lost and finally all types of cost for inventory are consider in the model. Also, we presented a mixed integer programming model and proposed an exact solution procedure in regard to Benders' decomposition method.

For future research we suggest three directions:

(a) We can study this model while Z_{ij} is probability and this parameter does not have deterministic value. For example, it has a distribution probability function and based on it, Z_{ij}gets value 0 or 1.

(b) Setting inventory capacity constraint is another development for this model.

(c) Applying the other ordering methods to calculate inventory costs in designing distribution networks.

(d) Aggregating this model with other assumption such as routing and considering time window can be an interesting development of the proposed model.

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