Journal of Optimization in Industrial Engineering 18 (2015) 37-46

A Mixed Integer Programming Formulation for the Heterogeneous Fixed Fleet Open Vehicle Routing Problem

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Received 27 January, 2014; Revised 21 February, 2015; Accepted 04 March, 2015

Abstract

The heterogeneous fixed fleet open vehicle routing problem (HFFOVRP) is one of the most significant extension problems of the open vehicle routing problem (OVRP). The HFFOVRP is the problem of designing collection routes to a number of predefined nodes by a fixed fleet number of vehicles with various capacities and related costs. In this problem, the vehicle doesn't return to the depot after serving the last customer. Because of its numerous applications in industrial and service problems, a new model of the HFFOVRP based on mixed integer programming is proposed in this paper. Furthermore, due to its NP-hard nature, an ant colony system (ACS) algorithm was proposed. Since there were no existing benchmarks, this study generated some test problems. From the comparison with the results of exact algorithm, the proposed algorithm showed that it can provide better solutions within a comparatively shorter period of time.

Keywords: Open Vehicle Routing Problem, Heterogeneous fleet, Ant Colony System, Exact Algorithm, Mixed Integer programming.

1. Introduction

In today's commerce world, freight transportation plays a significant role in logistics and supply chain management. It supports and makes most other social and economic activities possible. That is why many companies are giving special attention to the transportation costs of goods in order to minimize their expenses. Companies try to reduce transportation costs by using rational manners and effective tools. Consequently, Capacitated Vehicle Routing Problem (CVRP) and its versions have been receiving much attention by researchers and scientists (Yousefikhoshbakht et al., 2014). The CVRP is the basic version of the VRP, where all customers are delivery customers, the demands are known, all vehicles are identical and they belong to the same central depot. The imposed constraints are related to the capacity of the vehicles, may also be restricted in the total distance. It can travel and all customers must be served by a single route. In this problem, the objective is to find a set of delivery routes satisfying these requirements and giving minimal total travel cost. To make CVRP models more realistic and applicable, there are many varieties of the VRP obtained by adding constraints to the basic model. Examples of such extensions are VRP with Time Windows (VRPTW) (Tzeng et al., 1997), VRP with backhauls (VRPB) (Popovic, 1995), Stochastic VRP (SVRP) (Teodorovic et al., 1995), Multi-depot VRP (MDVRP) (Geetha et al., 2012), VRP with simultaneously Pickup and Delivery (VRPSPD) (Yousefikhoshbakht et al., 2014), Split Delivery VRP (Ozfirat et al., 2010), Open VRP (OVRP) (9) and so on with different constraints (Yousefikhoshbakht et al., 2012).

Nowadays, many enterprises contract their physical distribution tasks to third party logistics companies. These outsourcing carriers are paid on the basis of fixed costs and traveling distances of the 'for-hire' vehicles. Therefore, the vehicle starts at the depot and terminate at one of the customers after servicing the last customer on its route. This problem is regarded as an OVRP in which the route of each vehicle is a Hamiltonian path. At first sight, having open routes instead of closed ones looks like a minor modification. Indeed, if travel costs are asymmetric, there is essentially no difference between the open and closed versions. In other words, to transform the open version into the closed one, it suffices to set the cost to zero for traveling from any customer to the depot. However, if travel costs are symmetric, things are more subtle. Indeed, the open VRP turns out to be more general than the closed VRP, in the sense that any closed version

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own a heterogeneous fleet of vehicles available for hiring, considering a fleet of homogeneous vehicles is not practical in the VRP. Hence, the heterogeneous fleet vehicle routing problem has been investigated by many scientists and researchers (Li et al., 2010). Moreover, if in this problem, the number of vehicles available to perform the delivery is considered limited and the vehicles have different capacities, fixed costs and variable costs per unit distance, it is called heterogeneous fixed fleet open vehicle routing problem (HFFOVRP).

The HFFOVRP can be converted into an open vehicle routing problem (OVRP) and heterogeneous fixed fleet vehicle routing problem (HFFVRP). In more detail, the OVRP can be obtained from HFFOVRP by removing the constraint of vehicle heterogeneity. Similarly, HFFOVRP by removing the constraint of vehicle heterogeneity of Hamiltonian path can be converted to a VRP with a heterogeneous fixed fleet. Since OVRP and HFFVRP are NP-hard combinatorial optimization problems (Cao et al., 2010), The HFFOVRP is an NP-hard problem.

Recently, most of the research effort aimed at solving the NP-hard problems, have focused on the development of various meta-heuristic algorithms. A meta-heuristic can be defined as a top-level general strategy which guides other heuristics to search for good solutions in feasible space. Most of the VRP meta-heuristic algorithms are based on some construction and improvement heuristics, i.e., they use the so-called local search principle (Reimann et al., 2004). The aim of this paper is to apply the exact algorithm and ant colony system (ACS) to deal with the new variant of the VRPs, HFFOVRP. To reach this goal, first we propose a mixed integer programming named as node based formulation and then obtained results by the CPLEX 12.4 and ACS are compared together for some instances.

The rest of this paper is organized as follows. In Section 2, a proposed mixed integer model is described. In Section 3, the details of the proposed approach are introduced. Experimental evaluation of this algorithm is reported in Section 4. Finally, we report the computational results of the proposed algorithm based on the generated benchmark problems. In Section 5, we conclude this paper and discuss some possible research extensions in future work.

2. Problem Description and Formulation

2.1. Problem definition

From a graph theoretical point of view, we can define the HFFOVRP as follows. Let G = (V, E) be an undirected connected graph with $V=\{0,1,\dots,n\}$ as the set of vertexes and the set of arcs $E=\{(i,j):0\le i,j\le n\}$ (if the graph is not complete, we can compensate the lack of each arc with the arc that has infinite size). Node 0 is the depot and the customer set *C* consist of *n* customers, i.e. $C=\{1,2,\dots,n\}$. A nonnegative cost d_{ij} ($d_{ii}=0,\ 0\le i\le n$) associated with each arc $(v_i,v_j)\in E$. v_0 represents the depot and each vertex $v_i\in C$ is a customer with a non-negative demand p_i. The available fleet consists of K different type vehicles located at the depot and the number of available vehicles of each type is fixed and equal to n_k. A capacity Q_k, a fixed cost f_k, variable cost α_k is associated with each type of vehicle k and. α_k is cost per unit of distance corresponding to each vehicle type *k*. Hence, $c_{ij}^k=d_{ij}\times\alpha_k$ represents the cost of the travel from customer *i* to *j* with a vehicle of type *k*. The HFFOVRP deals with finding the minimum total transportation cost including the fixed and variable cost for a fleet of vehicles which start and end at the depot, so that the following constraints are taken into account:

- The total load of each vehicle cannot exceed the capacity of the corresponding vehicle type.
- The number of vehicles of type k used cannot exceed n_k.
- The demand of each customer is satisfied by exactly one vehicle in only one visit.

2.2. Problem formulation

We present following mathematical formulation for HFFVRP using variables and y_{ij} where, x_{ij}^k take the value 1 if a vehicle of type *k* travels directly from customer *i* to customer *j*, and 0 otherwise; denotes the route. The flow variables y_{ij} specify the quantity of goods that a vehicle *k* is carrying when leaves customer *i* to service customer *j*.

$$\text{Min}\sum_{k=1}^{K}f_k\sum_{j=1}^{n}x_{0j}^k+\sum_{k=1}^{K}\sum_{i=0}^{n}\sum_{j=0}^{n}c_{ij}^k x_{ij}^k \tag{1}$$

subject to

$$\sum_{k=1}^{K}\sum_{i=0}^{n}x_{ij}^k=1 \qquad \forall j=1,2,\dots,n \tag{2}$$

$$\sum_{k=1}^{K}\sum_{j=1}^{n}x_{ij}^k\le 1 \qquad \forall i=1,2,\dots,n \tag{3}$$

$$0\le\sum_{i=1}^{n}x_{ij}^k-\sum_{i=1}^{n}x_{ji}^k\le 1 \qquad \forall j=1,2,\dots,n,\ \forall k=1,2,\dots,K \tag{5}$$

$$\sum_{j=1}^{n}x_{0j}^k\le n_k,\qquad \forall k=1,2,\dots,K \tag{6}$$