Stochastic Vehicle Routing Problems with Real Simultaneous Pickup and Delivery Services

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Abstract

The problem of designing a set of routes with minimum cost to serve a collection of customers with a fleet of vehicles is a fundamental challenge when the number of customers to be dropped or picked up is not known during the planning horizon. The purpose of this paper is to develop a vehicle routing Problem (VRP) model that addresses stochastic simultaneous pickup and delivery in the urban public transport systems of Addis Ababa city Bus Enterprise, in Ethiopia. To this effect, a mathematical model is developed and fitted with real data collected from Anbessa City Bus Service Enterprise (ACBSE) and solved using Clark-Wright saving algorithm. The form-to-distance is computed from the data collected from Google Earth and the passenger data from the ACBSE. The findings of the study show that the model is feasible and showed an improvement as compared to the current performances of the enterprise. It showed an improvement on the current number of routes (number of buses used) and the total kilometer covered. The average performances of the model show that on average 6.48 routes are required to serve passenger demands of 271 and on average the simulation run was performed with 0.40 seconds of CPU time. During this instance, the average distance traveled by the vehicles in a single trip is 552.92kms.

Keywords: Stochastic VRP, Simultaneous Pickup and Delivery, VRP.

1. Introduction

VRP has been a well-studied combinatorial optimization problem for the last half of century. Many extensions have been evolved from it in order to support decision making under different real-world conditions. Due to this reason, various variants of VRP have been developed from it. Moreover, since the introductions of VRP, there have been many new insights and algorithms developed to study the classical or deterministic VRP (Kallehauge, 2006) as well as for the stochastic and dynamic variations of it (Bertsimas and David, 1996; Golden and Stewart, 1978; Ismail and Irhamah, 2008).

The variants of VRP models are differed by the inclusion or exclusions of different side constraints. The most common side constraints are restrictions on capacity of vehicle (Chenghua & Xiaofeng, 2011), total time or time windows (Madsen et al., 1995), precedence relations between pairs of cities and the number of depots (Paolo & Daniele, 2002; Laporte, 1992). The variant of VRP most commonly studied is VRP with Time Windows (VRPTW) (Cordeau et al., 2001; Madsen et al., 1995; Cordeau et al., 2007; Kenyon & P. David, 2003). Other variants of VRP are VRP with Pickup and Deliveries and Time Windows (CVRPPDTW), Multiple Depot VRP with Time Windows (MDVRPTW), Split Delivery VRP with Time Windows (SDVRPTW) and Periodic VRP with Time Windows (PVRPTW) (Dror & Trudeau, 1989; Dror et al., 1994; Dror & P, 1990); and VRP with Pickup and Delivery (VRPPD), where the delivery and pickup are treated separately as VRP with divisible Delivery and Pickup (Anbuudayasankar & Ganesh, 2008; Parragh et al., 2008).

In the area of stochastic environment, some of the variants are Stochastic VRP (SVRP) and/or VRP with stochastic Customer Demand (VRPSD), SVRP with pure pickup or a pure delivery problems (Bertsimas, 1992). This type of VRP has been attempted in the literature with areas of application such as model public bus transport (Eshetie et al., 2014b), waste collection (Milić & Jovanović, 2011), plant to customer distribution (Chepuri &Tito, 2005), collection and delivery of goods (Bertsimas, 1992), etc.. In the stochastic VRP environment, VRP with stochastic customer demand is the most studied variant in the literature (Eshetie et al., 2014a).

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$L_p(v_j) + p_p(v_{j+1}) - L_d(v_j)$ is the remaining net load in the bus between any consecutive nodes then the route will be feasible if $p_p(v_j) \le Q$.

To formulate the mathematical model, the following notations and representations are used.

Let $V = \{v_0, v_1, v_2, \ldots, v_n\}$ be a set of nodes or vertices treated as bus stops, and v_0 be a depot where buses will be dispatched.

- A is the set of arcs (i, j) A
- m is the number of vehicles $k = \{1, 2, \ldots, m\}$
- c_{ij} is the distance traversing the arc from node i to node j
- p_i is the expected number of passengers to be picked at node i,
- d_i is the expected number of passengers to be dropped at node i
- Q the vehicle capacity
- n total number of nodes or vertices or bus stops included in the model.
- y_i^k the expected cumulative number of passengers picked by vehicle k when leaving from node i, and $x_{ij}^k = 1$ if vehicle k travels from node i to node j; 0 otherwise.
- z_i^k the expected number of passenger/s which is/are remaining in vehicle k when leaving from node i
- $x_{ij}^k = 1$; if vehicle k travels from node i to node j; 0 otherwise

The objective is to determine a route for each vehicle that serves a set of nodes (v_j) so that the total distance traveled $\sum\sum c_{ij} x_{ij}$ is minimized. However, it is subjected to the following constraints. That is, each vehicle leaves the depot is used at most once, that is $\sum_{j=1}^{n} x_{0j}^k \le 1$; each node has to be served exactly by one vehicle and expressed as $\sum_{i=0}^{n} x_{ij}^k = 1$; the same vehicle arrives in node i must leave from same node is represented by $\sum_{i=0}^{n} x_{ij}^k - \sum_{i=0}^{n} x_{ji}^k = 0$. Moreover, $z_i^k + y_i^k \le Q$ which ensures that the expected cumulative net load on vehicle k when leaving from node i is always less than the vehicle capacity.

The traversed load constraint, $(z_i^k - d_j - z_j^k)\, x_{ij}^k = 0$ and $(y_i^k - p_j - y_j^k)\, x_{ij}^k = 0$, indicate that when arc (i, j) is traversed by vehicle k, the number of passengers to be dropped by the vehicle has to be decreased by d_j while the number of passengers picked up has to be increased by p_j, respectively. At the depot, the bus starts its service with full capacity; this indicates that $z_0^k = y_0^k = 0$. The overall model is then summarized and given as follows:

$$\text{Minimize} \quad \sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=0}^{n} c_{ij}^k x_{ij}^k$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{0j}^k \le 1$$

$$\sum_{k}\sum_{i=0}^{n} x_{ij}^k = 1$$

$$\sum_{i=0}^{n} x_{ij}^k - \sum_{j=0}^{n} x_{ji}^k = 0$$

$$z_i^k + y_i^k \le Q$$

$$(z_i^k - d_j - z_j^k)\, x_{ij}^k = 0$$

$$(y_i^k - p_j - y_j^k)\, x_{ij}^k = 0$$

$$z_0^k = y_0^k = 0$$

$$y_i^k, y_j^k \ge 0$$

$$z_i^k, z_j^k \ge 0$$

$$k = 1, 2, \ldots, m \quad \forall (i, j), \; i, j = 1, 2, \ldots, n$$

During the planning horizon, the minimum number of vehicles m can be determined using;

$$m = \max\left\{ \left\lceil \frac{\sum_{i=1}^{n} d_i}{Q} \right\rceil, \left\lceil \frac{\sum_{i=1}^{n} p_i}{Q} \right\rceil \right\}$$

3. Model Validation

To validate the model given in section 2, the real scenario of ACBSE operations are modified and fitted on ACBSE. It is the only urban public bus transport service enterprise in the city of Addis Ababa, Ethiopia. Currently, it serves more than 115 routes that connect different parts of the city using 690 operational buses (Eshetie et al., 2013b). However, for the purposes of this research, only one depot that has 58 bus stops are considered. On this depot, on average, the company uses 11buses and covers a 752Km in a single trips to serve the 58 bus stops. The following terms and representations are used during the model fitting processes:

Passengers and depot: $V = \{v_0, v_1, v_2, \ldots, v_{58}\}$ is a set of nodes including the depot corresponding to the locations of passengers to be picked or dropped.

Demands: Passengers have stochastic pickup and delivery demands. Actual passengers' demand of each location v_i is only known when the vehicle arrives at passengers' location i.

Vehicles capacity: The vehicle has a limited capacity of $Q = 70$ passengers. If the total expected passengers (i.e. the cumulative of the difference between dropped and picked up) exceeds the vehicle capacity, route failure is said to occur. But no penalty cost is assumed in this case. The bus terminates its services and gets back to the depot.

Route: A route must start at the depot, visit a number of passengers' locations or bus stops and return to the depot. A feasible route to the SVRPSPD is a permutation of the bus stops $R = (v_0, v_1, v_2, \ldots, v_n)$

starting and ending at the depot (ὸ ὤὸ ῏ί, ὺ$_{ي}$ = ὺ$_{ح}$ = 0) and it is called a priori tour.

Distance to be minimized: let ὂ = (῎, ῏): ῎, ῏ ὠ, ῎ ≠ ῏ is the set of arcs joining the nodes and a non-negative matrix ὅ = {ὢ$_{ἰح}$: ῎, ῏ ὠ, ῎ ≠ ῏} denotes the traveling distances between nodes ῎ and ῏. The from-to-distance of a sample of origin-destination are computed from the digital latitude and longitude data from Table1 using great circle computation and presented in Table2.

3.1 Input Parameters to the Model

To run and test the model, the input parameters that have to be fitted to the model are required. These inputs are either collected or generated/computed. These data are the from-to-distance, the passengers demand and the demand distributions which are computed or generated at each location point. Each of them are briefly explained and presented in section 3.1.1 and 3.1.2.

3.1.1 Stochastic Passengers Demand

The passengers' demand collected in 58 location points of ACBSE from 2004 to 2014 are used to fit the demand behavior of passengers. The snapshot of passengers picked up (ἡ$_{ἰ}$) and passengers dropped (Ὠ$_{ἰ}$) and the digital location point ὺ$_{ἰ}$ are reported in Table1.

3.1.2 From-to-Distance

The from-to-distance for each location ῎ is computed by taking the longitude and latitude locations of each point using Great Circle distance formula that considers the circular nature of earth. Due to the symmetrical nature of the data, the upper triangular matrix of the from-to-distance is reported in Table2. Each ὢ$_{ἰح}$ is defined as the distance from ῎ to ῏, which can be directly considered as the cost associated to transport passenger including depot 0. Further, it assumes the distance is symmetric, that is ὢ$_{ἰح}$ ὢ$_{حἰ}$ and ὢ$_{ἰἰ}$ ὢ$_{حح}$ = 0, and satisfies the triangular inequality.

Table 1
Sample demand distribution at each location

ὺ$_{ἰ}$	*Location (decimal)*		*Expected Passengers*	
	Longitude	*Latitude*	ἡ$_{ἰ}$	Ὠ$_{ἰ}$
0	38.8	9.16	-	-
1				
2	38.9	9.1	18.42	12.12
3	38.9	8.94	8.35	11.40
4	38.8	9.16	16.83	11.55
5	38.9	9.13	8.50	4.29
6	38.8	9.04	5.58	8.98
7	38.8	9.18	17.17	8.91
8	38.8	9.14	17.93	12.94
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
57	38.6	9.11	14.72	14.93
58	39.0	9.13	12.42	12.00
59	38.8	9.04	11.56	4.99

Table 2
From-to-distance matrix (ὢ$_{ἰح}$)

ὺ$_{ἰح}$	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	.	.	.	*56*	*57*	*58*
0	-		17	7	7	8	1	1	.	.	.	17	20	19
1		-	11	7	2	8	9	7	.	.	.	22	14	23
2			-	11	13	10	18	15	.	.	.	33	20	21
3				-	8	1	8	6	.	.	.	24	21	15
4					-	9	8	7	.	.	.	20	14	24
5						-	10	7	.	.	.	26	22	15
6							-	3	.	.	.	16	21	20
7								-	.	.	.	19	20	18
.	.	.	.	.	.	.	.	.	-	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	-	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	-	.	.	.
56												-	27	34
57													-	37
58														-

3.2 Solving the Model

A heuristic procedure developed for the classical VRPs has been extended to solve SVRPSPD model developed above. The heuristics adopted is Clarke-Wright algorithm. The Clarke-Wright algorithm is iteratively repeated for each node as a starting node with the objective of improving the quality of the solution. The following assumptions are considered during solving of the model.

- All routes start and end at the depot.
- Each node in ὺ is visited exactly once by a vehicle.
- The cumulative demand along the route or demand at any node shall never exceed the vehicle capacity Ữ.
- All vehicles have the same capacity and are stationed at the node of origin.
- Split delivery is not permitted.
- Each vehicle makes exactly one trip during the simulation run.

The task is to determine a route for each vehicle so as to serve a set of nodes so that the total distance traversed is minimal. The solution using a Clarke-Wright algorithm is obtained using the following steps:

- Step 1. Select any node from the central depot ὺ.
- Step 2. Compute savings using ί = ὦ + ὦ − ὦ or ῗ, ῗ 2, 3 , ... , ὲ.
- Step 3. Order the savings from largest to smallest.
- Step 4. Start at the top of the savings list and move downward, form larger sub-tours by linking appropriate nodes ῗ and ῗ and determine the net passengers' demand, Ữ(ὺ), until the vehicle attains its maximum load Ữ. If Ữ(ὺ) ≤ Ữ, the tour is feasible. That is the total cumulative demand, Ữ(ὺ) should be less than or equals to the vehicle capacity Ữ. Otherwise the model needs to define a new tour in which node, ὺ will be the starting node and passenger's demand, Ữ(ὺ) takes the first position.
- Step 5. Insert the depot between the first and the last nodes of the tour.
- Step 6. Repeat from step 2 until all nodes are visited or all passengers are serviced.

Since the developed model is new, most of the time, the worst behavior in such an algorithm is not known. However, according to Golden and Stewart (1978), for a sequential version of such an algorithm where at each step selecting the best savings from the last node added to the sub-tour the worst case ratio is bounded by a linear function in ᾶῺ(ὲ) that is a running time of 5.88. Thus, Clarke and Wright savings procedure for this model requires the order of ὲ ᾶῺ computation, which is about 6164.336 computational time.

Using Clarke-Wright savings algorithm, the route is constructed by incrementally selecting passengers along the nodes until the cumulative number of passengers reaches the vehicle capacity or all customers are visited. Initially, each vehicle starts at the depot ὺ empty and set passengers included in the tour. The algorithm selects the next customer to visit from the list of feasible locations and the capacity of the vehicle is updated before the next location is selected and included in the tour. The vehicle returns to the depot when the capacity constraint of the vehicle is met or when all the passengers at each location are visited. Finally, the total minimum distance ∑ὦ ὠ is computed as the objective function value for the complete route of the vehicle. For each route, the total demand Ữ(ὺ) and the CPU computation time are also computed.

The findings of the simulation result are summarized and shown in Table 3. The findings show that the minimum and maximum distance covered is 432 and 646. The maximum CPU time recorded is 2.27 seconds which is the worst case, whereas the minimum CPU time is 0.13 seconds. The average performances of the model show that on average 6.48 routes are required to serve passenger demands of 271 and on average the simulation run was performed with 0.40 seconds of CPU time. During this instance, the average distance traveled by the vehicles in a single trip is 552.92ὺά. As compared to the current performances of the ACBSE stated on section3, the findings of the simulation run show that there is an improvement of 26.47% on the kilometer coverage and 41.09% on the number of trips (buses) involved during the service of the 58 bus stops.

Table 3
Summary output of the 25 Run

Run	Total Demand	Total Distance (Km)	Number of Routes	CPU Time
Min	179	432	5	0.13
Max	427	646	8	2.27
Mean	271.20	552.92	6.48	0.40
Stdv.	49.49	61.09	0.77	0.42

4. Conclusions

The developed model is the first of its kind due to the fact that it considers VRP with real simultaneous pickup and delivery at each bus stop and treats them as stochastic and random. The findings of the solution show that the model developed and the solution achieved at this level are feasible to be considered as outputs to the solution space. As compared to the current bus scheduling performances of ACBSE, the findings of the study show an improvement on the number of trips made (buses involved for each trip) and total kilometer covered during a single scheduling horizon.

Moreover, the connected routes in the solution space may not be the practical routes that existed in the city. Thus, this should be considered as a limitation of the model, which is to be addressed as further research directions in the in future.

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