Monitoring Process Variability: A Hybrid Taguchi Loss and Multi-objective Genetic Algorithm Approach

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Abstract

The common consideration on economic model is that there is knowledge about the risk of occurrence of an assignable cause and the various cost parameters that does not always adequately describe what happens in practice. Hence, there is a need for more realistic assumptions to be incorporated. In order to reduce cost penalties for not knowing the true values of some parameters, this paper aims to develop a bi-objective model of the economic-statistical design of the S control chart to minimize the mean hourly loss cost while minimizing out-of-control average run length and maintaining reasonable in-control average run length considering Taguchi loss function. The purpose of Taguchi loss function is to reflect the economic loss associated with variation in, and deviations from, the process target or the target value of a product characteristic. In contrast to the existing modeling approaches, the proposed model and given Pareto-optimal solution sets enables the chart designer to obtain solutions that is effective even for control chart design problems in uncertain environments. A comparison study with a traditional economic design model reveals that the proposed chart presents a better approach for quality system costs and the power of control chart in detecting the assignable cause.

Keywords: Economic-Statistical design, Taguchi loss function, NSGA-II Algorithm, Process variability, Immeasurable costs.

1. Introduction

When a production process faces with an assignable cause, it may shift the process variance to out-of-control states. Factors such as faculty (variable quality), raw material, unskilled/careless operators, and loosening of machine settings may lead to an increase in process variability without necessarily influencing the level of the process mean (Collani and Sheil, 1989). The *S* control chart is useful for monitoring a change in the process variance.

The usual approach in economic design of control charts is to develop a cost model for a particular type of industrial process and finding the optimal parameters that minimize the long-run expected cost per hour using the optimization methodologies. Several researchers studied the economic design of control charts (Collani and Sheil, 1989, Yeong et al., 2013, Faraz and Saniga, 2013, Asadzadeh and Khoshalhan, 2009, Amiri et al., 2014, Safaei et al., 2015). However, the supposition that everything is all right inside the specification and all wrong outside does not correspond to this world. Deming (1982) believes that the Taguchi loss function in which there is a minimum loss at the nominal value and an ever-increasing loss with departure either way from the nominal value is a better description of the world.

Economic design model involving Taguchi's loss function takes the advantages of both Taguchi concepts and SPC in which any deviation from the target is penalized whether the system is in-control or out-of-control (Safaei et al., 2015). Taguchi *et al.* (1989) provided an economic design to determine the diagnosis interval and control limits. The loss function as a rational approach for the minimization of the process variation has been widely studied. Yang (1998) developed the first economic statistical design of *S* chart which embellished with Taguchi loss function. Several researchers have applied the loss function approach in the economic design of control charts (see, e.g., ((Ben-Daya and Duffuaa, 2003); (Jiao and Helo, 2008)). Using the multivariate Taguchi loss function approach, Niaki *et al.* (2010) recently extended the economic and economic-statistical models of the MEWMA control chart in monitoring the mean vector of a process.

In reality, a certain function exists for each quality characteristic that uniquely defines the relationship between the economic loss and the deviation of the quality characteristic from its target value. The concept involved in Taguchi methods is that useful results must be obtained quickly and at low cost. Use of a quadratic,

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$$ARL_0 = 1/\left[1 - G((n-1)L_S^2)\right] \quad (5)$$

and the out-of-control ARL is

$$ARL_1 = 1/\left[1 - G((n-1)L_S^2/(\sigma_1^2/\sigma_0^2))\right] \quad (6)$$

3. Multi-objective S Chart

The primary assumption in economic models of control charts is that various cost parameters and the risk of occurrence of assignable causes are known (Pignatiello and Tsai, 1988, Reynolds and Cho, 2006, Chen and Liao, 2004). However, some cost parameters are difficult to accurately estimate in practice. For instance, hourly cost due to nonconformities produced while the process is out-of-control (C_1) is difficult to estimate because it involves an immeasurable diminishment in customer goodwill, besides measurable freight and indemnity. Similarly, the cost of investigating a false alarm a'_3 also involves immeasurable portion (Chen and Liao, 2004).

Moreover, Multicriteria optimization algorithms are studied in economic design of control chart. Celano and Fichera (1999) developed an $\bar{X}$ chart considering the optimization of the costs and at the same time the statistical proprieties, whereas in the multiobjective configuration the fitness is the expected loss per hour multiplied by a coefficient function of the weighed sum of α and (1-β). Bakir and Altunkaynak (2004) applied such an algorithm to develop $\bar{X}-R$ chart. Asadzadeh and Khoshalhan (2009) developed a procedure to derive a multiobjective decision-making model with multiple assignable causes.

Recently, Safaei *et al*. (2012) formulated an optimal design of $\bar{X}$ control chart as a multiple objective decision-making problem. Faraz *et al*. (2013) studied a multiobjective Genetic Algorithm Approach to the Economic Statistical Design of Control Charts with an Application to $\bar{X}$ and S^2 Charts. Morabi *et al*. (2015) considered a multiobjective design of $\bar{X}$ control charts with fuzzy process parameters.

The optimal design of an S control chart involves determining the optimal values of the chart parameters, i.e., the sample size, the interval between successive samples, and the control limit.In this study, a model that considers both measurable and immeasurable costs in a multiobjective economic-statistical design of control chart is considered. Correspondingly, Minimization of ARL_1 is cooperated as quality performance indices in the model to give another consideration to the aspect of immeasurable costs.

Let T be the target value for the quality characteristic monitored the quality loss is zero only when the quality characteristic X equals the target T. The loss increases as the deviation from the target increases. Suppose that μ_0 and σ_0 are in-control mean and standard deviation, respectively. When the process goes to an out-of-control situation, the process variance, becomes $\sigma_1^2=\rho^2\sigma_0^2$ ($\rho \geq 1$). Further, Assume also the probability density function (pdf) of the quality characteristic X to be $f(x)$. If the loss function, $L(X)$, is symmetric around the target, the loss coefficient K should be estimated such that the loss can be obtained as

$$L(X) = K(x-T)^2 \quad (7)$$

To design control charts based on a symmetric quadratic loss function, J_0 is calculated as

$$J_0 = \int_{-\infty}^{+\infty} K(x-T)^2 f(x)dx = \int_{-\infty}^{+\infty} K(x-\mu_0+\mu_0-T)^2 f(x)dx = K\left[\sigma_0^2 + (\mu_0-T)^2\right] \quad (8)$$

Where $f(x)$ is probability density function (pdf) of a normal variable $N(\mu_0,\sigma_0^2)$.

The expected cost per unit under quadratic loss function when the process is out-of-control is

$$J_1 = K\left[\sigma_1^2 + (\mu_0-T)^2\right] \quad (9)$$

The expected external costs of each product are shown by J_0 and J_1. If p units are produced per hour, C_0 and C_1 in cost function can be computed as, $C_0=J_0p$ and $C_1=J_1p$.

The optimal values of the control parameters are determined such that the ATL of the company is minimized and the detection power of the chart is maximized. In other words, the multiobjective economic-statistical model for S chart becomes:

$$\begin{aligned} & Min\ ATL(L_S,n,h) \\ & Min\ ARL_1 \\ & Subject\ to \\ & \quad ARL_0 > ARL_L \\ & \quad n\ is\ a\ positive\ integer \\ & \quad h\ and\ L_S > 0 \end{aligned} \quad (10)$$

In the next section, a solution algorithm is proposed to solve the multiobjective optimization model given in (10).

4. The Solution Algorithm

The economic models are of nonlinear programming (NLP) type with linear constraints and there are different methods available in literature to solve such models. Molnau *et al*. (2001) used the algorithm of Hooke and Jeeves (1961) to solve the nonlinear programming model of economic design of control charts. However, since one of the model parameters, L_S, is indirectly used in the model and is solely used for calculation of ARLs, a search-based algorithm (for example genetic algorithm) can be applied to solve the model. These algorithms by searching different quantities of decision variables and using appropriate penalty or barrier functions can converge to the best solutions.

The optimal *S* chart parameters minimizing both the *ATL* and ARL_1 are given in Table (1). The Pareto front for *ATL* and ARL_1 of the multi objective economic-statistical design is shown in Figure (1).

From the economic viewpoint, the solution vector at minimum *ATL* is (1.60,9,1.54) with $344.68 with an ARL_1 of 1.34. However, a slight movement from the minimum *ATL* ($0.3), as revealed in Table (1) and Figure (1), solution (1.55,11,1.74) results improvement in the both statistical properties with ARL_1 of 1.22 and ARL_0 142.4 where it receives higher product quality. Accordingly, this multiple objective design of *S* control chart present a better approach for quality engineers to improve the process.

Table 1
Optimal design of *S* chart

L_S	*n*	*h*	ATL	ARL_1	ARL_0
1.60	9	1.54	344.68	1.34	130.6
1.60	9	1.54	344.68	1.34	130.6
1.59	9	1.55	344.68	1.33	123.5
1.59	9	1.55	344.68	1.33	123.5
1.57	10	1.64	344.77	1.27	125.7
1.57	10	1.64	344.77	1.27	125.7
1.56	10	1.76	344.81	1.26	105.7
1.55	11	1.74	344.98	1.22	142.4
1.53	12	1.86	345.29	1.18	127.0
1.53	12	1.86	345.29	1.18	127.0
1.53	12	1.86	345.29	1.18	126.5
1.50	13	2.00	345.68	1.14	127.1
1.50	14	2.00	346.11	1.13	146.7
1.49	15	2.08	346.60	1.11	167.0
1.45	15	2.15	346.69	1.09	121.1
1.46	18	2.32	348.29	1.07	265.3
1.45	19	2.53	348.97	1.05	237.4
1.38	21	2.49	350.51	1.02	105.0
1.37	26	3.31	354.10	1.01	200.6
1.37	27	4.36	356.67	1.01	237.0
1.32	28	5.68	360.99	1.00	102.8
1.32	28	5.68	360.99	1.00	102.3

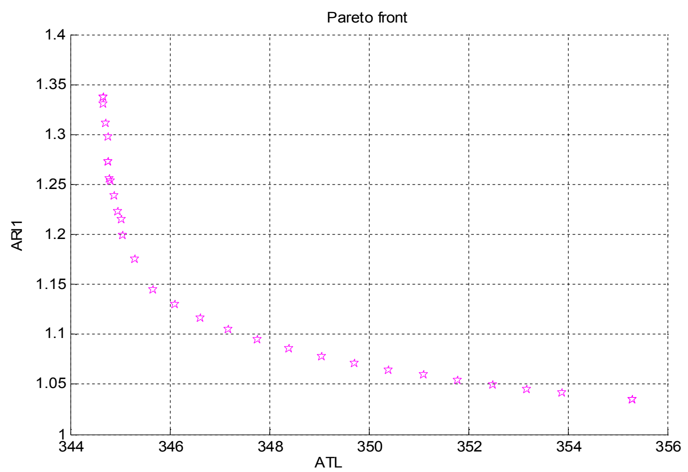

Fig. 1. Pareto front for *ATL* and ARL_1

5.2. *Sensitivity Analysis*

In this section, the sensitivity of the chart parameters to variations in process parameters is explored. The parameters are set as follow. Two values for cost per false alarm, $a'_3 \in \{300,900\}$, and two values for cost to locate and repair the assignable cause, $a_3 \in \{150,900\}$, are considered. Moreover, the process variance shifts are of the sizes of ρ=1.5, 2, 2.5. Figure (2) summarize the Pareto front for 12 runs of the parameter values. In Figure (2), a_3= $150 is shown in purple, a_3= $900 is exposed by blue color, and three iso-chromatic curve in the chart are based upon the sizes of process shift (small, moderate and large).

Table 2
The results of comparison study with Serel (2009) *S* chart

Multiobjective economic-statistical design of *S* chart s.t. $ARL_0 \geq ARL_L$								Traditional *S* chart		
a'_3	a_3	ρ	L_S	n	h	ATL	ARL_1	ATL	ARL_1	ARL_0
300	150	1.5	1.35	16	2.92	331.40+	1.5	331.40	1.5	38.3
			1.34	17	3.04	331.41	1.4			
		2	1.6	9	1.54	344.68*	1.3	344.71	1.3	105.1
			1.55	11	1.74	344.98	1.2			
		2.5	1.78	7	1.09	361.84*	1.2	361.85	1.2	241.2
			1.69	9	1.27	362.84	1.1			
	900	1.5	1.34	17	3.09	338.50	1.4*	338.50	1.5	38.3
			1.32	20	3.39	338.65	1.3			
		2	1.62	9	1.49	351.95*	1.4	351.96	1.4	139.7
			1.58	10	1.61	352.01	1.3			
		2.5	1.78	7	1.1	369.11*	1.2	369.12	1.2	241.2
			1.69	9	1.27	370.10	1.1			
900	150	1.5	1.3	22	3.26	334.08*	1.5	334.10	1.5	115.8
			1.37	23	3.31	334.11	1.4			
		2	1.66	10	1.54	346.51*	1.4	346.53	1.3	328.3
			1.63	11	1.64	346.52	1.3			
		2.5	1.88	7	1.05	363.27*	1.3	363.30	1.3	593.7
			1.82	8	1.15	363.40	1.2			
	900	1.5	1.37	22	3.3	341.14*	1.5	341.15	1.5	115.8
			1.36	23	3.32	341.16	1.4			
		2	1.66	10	1.54	353.73	1.4	353.75	1.3	328.3
			1.63	11	1.65	353.74*	1.3			
		2.5	1.88	7	1.06	370.54*	1.3	370.57	1.3	593.7
			1.83	8	1.15	370.66	1.2			

Comparisons with traditional economic design model reveal that the proposed multi objective model is superior to the traditional models.

The model allows the easy and fast optimization of quality costs and statistical performance measures. The resulting Pareto-optimal solution set can be used to extract knowledge that could not be determined using single objective approaches, including the trade-off relationships between *ATL* and ARL_1. This provides a variety of choices to arrive at the requirement of long run quality of product or minimal cost concurrently for quality engineers. It is interesting to extend the current work for other control charts or to consider other loss function policies for the construction of the total cost model.

References

Amiri, F., Noghondarian, K. & Safaei, A. S. (2014). Evaluating the performance of variable scheme X-bar control chart: a Taguchi loss approach. International Journal of Production Research, 52, 5385-5395.

Asadzadeh, S. & Khoshalhan, F. (2009). Multiple-objective design of an $\bar{X}$ control chart with multiple assignable causes. The International Journal of Advanced Manufacturing Technology, 43, 312-322.

Bakir, M. A. & Altunkaynak (2004). The optimization with the genetic algorithm approach of the multi-objective, joint economical design of the $\bar{x}$ and R control charts. Journal of Applied Statistics, 31, 753-772.

Ben-daya, M. & Duffuaa, S. O. (2003). Integration of Taguchi's loss function approach in the economic design of x chart. International Journal of Quality & Reliability Management, 20 607-619.

Celano, G. & Fichera, S. (1999). Multiobjective economic design of an $\bar{X}$ control chart. Computers & industrial engineering, 37, 129-132.

Chen, Y. K. & Liao, H. C. (2004). Multi-criteria design of an X control chart. Computers & Industrial Engineering 46, 877-891.

Collani, V. & Sheil, J. (1989). An approach to controlling process variability. Journal of Quality Technology, 21, 87-96.

Deb, K. (2001). Multi-objective Optimization Using Evolutionary Algorithms, Chichester, UK, John Wiely.

Deming, W. E. (1982). Out of the Crisis, Cambridge, MIT Press.

Faraz, A. & Saniga, E. (2013). Multiobjective Genetic Algorithm Approach to the Economic Statistical Design of Control Charts with an Application to X¯bar and S2 Charts. Quality and Reliability Engineering International, 29, 407-415.

Hooke, R. & Jeeves, T. (1961). Direct search solution of numerical and statistical problems. J Assoc Comput Mach, 8, 212-229.

Jiao, J. & Helo, P. (2008). Optimization design of a CUSUM control chart based on taguchi's loss function. The International Journal of Advanced Manufacturing Technology, 35, 1234-1243.

Lorenzen, T. J. & Vancle, L. C. (1986). The Economic Design of Control Charts: A Unified Approach. Technometrics, 28, 3-10.

Mcwilliams, T. P., Saniga, E. M. & Davis, D. J. (2001). Economic-statistical design of $\bar{X}$ and R or $\bar{X}$ and S charts. Journal of Quality Technology, 33, 234-241.

Molnau, W. E., Montgomery, D. C. & Runger, G. C. (2001). Statistically constrained economic design of the Multivariate Exponentially Weighted Moving Average