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Research Article

A new Version of Earned Value Analysis for Mega Projects Under Interval-Valued Fuzzy Environment

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Abstract

The earned value technique is a crucial and important technique in analysis and control the performance and progress of mega projects by integrating three elements of them, i.e., time, cost and scope. This paper proposes a new version of earned value analysis (EVA) to handle uncertainty in mega projects under interval-valued fuzzy (IVF)-environment. Considering that uncertainty is very common in mega projects' activities, the proposed IVF-EVA model is very useful and applicable in evaluating the progress of projects. In this paper, analyzing earned value indices and calculating them with linguistic terms have been discussed. They are then converted into interval-valued fuzzy numbers (IVFNs) for the evaluations. Finally, an application example from the recent literature is presented and steps of the proposed IVF-EVA are elaborated.

Keywords: Earned value analysis; mega projects; interval-valued fuzzy sets; interval-valued earned value indices

1. Introduction

For a project, cost is an extremely important element of project management to run it successfully. The management of costs is commonly reflected in company's strategic goals, mission statements and business plan of a project organization in many ways (Pinto, 2010). In fact, project management is about how to apply a large number of resources to accomplish a unique, complex, one-time task within time, cost and quality constraints (Atkinson, 1999).

EVA is one of the project management's techniques for measuring the performance and progress of mega projects. According to Project Management Institute (PMI) when properly applied, the EVA provides an early warning of performance problems. EVA provides crucial information about performance and progress of mega projects for managers by studying the interaction of time, cost and scope. In fact, EVA calculates the cost and time performance indices of each project, and estimates the completion cost and the completion time of each project, and finally measures the performance and the progress of each project by comparing the planned value and actual costs of activities to their corresponding earned values (Naeini et al., 2013). Further, the EVA is a very helpful tool when predicting the outcome of the project. To make accurate predictions, it is very important that the plan can be of good quality. If the plan is not credible the same thing will be evident for the predictions. Hence, project managers really have to develop a plan that they do believe is possible to keep up with(Fleming and Koppelman,2005).

One of the first comprehensive guides on the earned value (EV) has been published in 2005 (PMI, 2005). Nowadays, it is accepted that using EV techniques has many advantages and would increase the schedule and cost performances of a project, although the studies on the EVA is restricted (Naeini et al., 2013). Lipke (1999) expanded cost and schedule indices to control the cost and schedule performances in a project. Lipke (2003) also introduced earned schedule (ES)'s concept. Henderson (2003, 2004) investigated the appropriation and validity of the ES. Anbari (2003) improved effectiveness of using EV. Kim et al. (2003) focused on the implementation of the EV in different sorts of firms and projects. Lipke (2004) improved cost and time performance probabilities of projects. Cioffi (2006) introduced a new notation for the EVA to make the EV calculations more clear and adjustable. Lipke et al. (2009) presented a reliable forecasting method of the final duration and cost in

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order to develop the ability of project managers to make right decisions.

Narbaev and De Marco (2013) concluded that the best and the most reliable method to estimate cost at completion was the ES-based method. Mortaji et al. (2013) formulated EVM using L-R fuzzy numbers. Colin and Vanhoucke (2014) implemented tolerance limits for statistical project control using EVM. Hunter et. al. (2014) improved cost monitoring and control through the earned value management system. Salari et. al. (2014) presented a fuzzy earned value model and provided estimation for a project total cost under uncertain conditions. Rubio et al. (2015) used EVM with performance indices evaluation and statistical methods to manage the project assessment. Willems and Vanhoucke (2015) presents an overview of the existing literature on project control and earned value management. Bastelier and Vanhoucke (2015) evaluated the accuracy for the time and cost forecasting of the projects.

To handle uncertainty in engineering and management systems, fuzzy sets theory presented separately by Grattan-Guinnes (1975), Jahn (1975), Sambuch (1975) and Zadeh (1975) in the mid-seventies. Through the years, necessityfor improving fuzzy sets theory arose as it was more applied in real-world situations. Interval-valued fuzzy Sets (IVFSs) is an growingly popular addendum of fuzzy sets theory where traditional [0,1] valued membership degrees are exchanged with intervals in [0,1] that approximate the exact degrees. Hence, not only vagueness but also a characteristic of uncertainty can be addressed intuitively.

In this paper, we introduce the uncertainty concepts with IVF-point of view. Considering most of activities in real-world applications for mega projects have a high degree of uncertainty in the EVA calculations. Taking account of uncertainty to the EVA makes results of calculations and final estimations of each mega project getting closer and reduces the errors in calculations. This paper studies the calculations of EV indices under uncertainty with linguistic terms and then transformed into IVF-numbers. Further, a new version of classical EVA with IVF-setting is proposed to deal with uncertainty in mega projects.

The remaining of this paper is organized as follows. Section 2 introduces the EV and its techniques. Section 3 expresses the IVF-sets theory and its application to the EV in a nut shell. Section 4 illuminates the developing of proposed new interval-valued fuzzy-based EV model and its interpretations. For clarification aims, an application example from the recent literature is discussed in details in Section 5. The paper ends with conclusions.

2. Basic Definition of Earned Value Analysis

The EV is physical work completed to date and the authorized budget for that work and designate how efficiently the project team deploys the project resources. The EV could be composed of some techniques to help project managers to evaluate and control the performance and progress of mega projects by estimating their completion cost and completion time based on its actual cost and actual time up to any given point the projects. Some of these techniques that can be mentioned are weighted milestone, fixed formula, percent complete estimates, percent complete and milestones, equivalent completed units, apportioned relationships and level of effort. The technique that is being used in this paper is percent complete technique. The percent complete technique method is a subjective judgment of the completed work. The person in charge of the work packages simply estimates the percent of work completed against the full planned value of that work package. This method is the commonly-used approach which is probably why it has received increasingly wide acceptance in the private industry (Fleming and Koppelman, 2005). However, it has obvious disadvantages, for example, it can sometimes be difficult to actually know how much work has been accomplished and as another instance uncertainty itself could cause biased judgments. An idea to conquer this problem is to use the linguistic terms and new extension of fuzzy sets theory in estimating the completion percent of each activity.

It is widely admitted that there exists some problems for which the solution acquired using fuzzy sets theory and related techniques sometimes are good in practice by representing the information provided by decision makers in mega projects. Notably in decision-making problems, according to the level of knowledge, experts might explain their preferences implementing exact numerical values. Distinctly, there are cases when the experts' level of knowledge about the environment, where the decision making is to be applied, is rather low or indefinite.

There are many situations in real-life mega projects where the amount of work (or the quantity of work) for an activity is unknown or imprecise, and is out of control. For example, in oil and gas projects, the amount of drilling process needed to be carried out per day is unknown. Oil may not be correctly located on the initial drilling plan. Other examples come from the field of construction projects like in a dam construction project the ground should be excavated until hard layer of rock is reached. Before reaching this layer, the exact amount of the operations and the required work are unknown, and also this is out of our control, so the percent complete of excavation activity cannot precisely be evaluated. In medical research projects and drug development projects, a majority of resources assigned to the clinical experiments aims at examining the new drugs for its benefits and potential side effects. The exact amount of work needed to derive scientific conclusions is unknown in advance. In these cases and many other similar cases, it would be better and easier to measure the percentage of the activity completed under uncertain values by linguistic terms with new extension of fuzzy sets theory rather to measure it exactly and deterministically. Some researchers believe the developed techniquesand models reflects better the uncertain nature of a mega project.

3. Concepts and Definitions of Interval-Valued Fuzzy Sets

Fuzzy sets theory introduced by (zadeh,1965) describes uncertainty and vagueness in events and systems. It is logical to model and consider the uncertainty by implementing linguistic terms with the fuzzy sets theory. These linguistic terms cannot be used on the EVA techniques before altering them to a number. Thus, firstly the fuzzy principles on the linguistic terms could be applied to transform them into IVF-numbers. Then, the EVA's mathematics will be adjusted to reflect the new values. Generally, project experts accomplish this transformation in accordance with their knowledge and experiences.

3.1. Interval-valued fuzzy numbers

This paper considers the fuzzy demand by applying IVF-sets. An IVF-set $\widetilde{A}$ defined on $(-\infty,+\infty)$ is given by Gorzaczany(1987) (Eq. (1):

$$\tilde{A} = \{x, [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)]\},\ x \in (-\infty,+\infty),\ \ \mu_{\tilde{A}^L}, \mu_{\tilde{A}^U} : (-\infty,+\infty) \to [0,1], \quad (1)$$
$$\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)],\ \ \mu_{\tilde{A}^L}(x) \le \mu_{\tilde{A}^U}(x),\ \forall x \in (-\infty,+\infty),$$

Where $\mu_{\tilde{A}^L}(x)$ and $\mu_{\tilde{A}^U}(x)$ are the lower and upper limits of the degree of membership (Kou,2011).

3.2. Interval-valued trapezoidal fuzzy numbers

From Chen (2006) and Wang and Li (2001), it can be seen that an interval-valued trapezoidal fuzzy number $\widetilde{A}$ can be represented by $\tilde{\tilde{A}} = [\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U] = [(a_1^L, a_2^L, a_3^L, a_4^L;\ w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U;\ w_{\tilde{A}^U})]$, as shown in Fig. 1 , where, $0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1$, $0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1$, $0 \le w_{\tilde{\tilde{A}}^L} \le w_{\tilde{A}^U} \le 1$, $w_{\tilde{A}^U} \ne 0$ and $\tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U$.
From Fig. 1, it can be seen that the IVF-numbers $\tilde{\tilde{A}}^L$and $\tilde{\tilde{A}}^U$, where $\tilde{\tilde{A}}^L$ is called the "lower trapezoidal fuzzy number" and $\tilde{\tilde{A}}^U$ is called the "upper trapezoidal fuzzy number"(Chen and Chen, 2009).

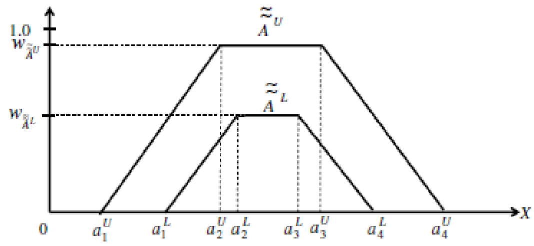


Fig. 1. An interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$

According to Fig. 1, the relations can be obtained as follows:

(1) If $\tilde{\tilde{A}}^L = \tilde{\tilde{A}}^U$, then the trapezoidal interval-valued fuzzy $\tilde{\tilde{A}}$ is a generalized trapezoidal fuzzy number.
(2) If $a_1^L = a_1^U = a_2^L = a_2^U = a_3^L = a_3^U = a_4^L = a_4^U$ and $w_{\tilde{\tilde{A}}^L} = w_{\tilde{A}^U}$, then the trapezoidal interval-valued fuzzy number $\tilde{\tilde{A}}$ is a crisp value.

The arithmetic operations between the IVF-numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are as follows:

(1) Interval-valued fuzzy number addition $\oplus$ (Eq. (2)):

$$\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L;\ \hat{w}_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U;\ \hat{w}_{\tilde{A}^U}\right)\right] \oplus \left[\left(b_1^L, b_2^L, b_3^L, b_4^L;\ \hat{w}_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U;\ \hat{w}_{\tilde{B}^U}\right)\right] = [(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L ; \min\left(\hat{w}_{\tilde{A}^L}, \hat{w}_{\tilde{B}^L}\right)), (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U ;\min (\hat{w}_{\tilde{A}^U}, \hat{w}_{\tilde{B}^U}))] \tag{2}$$

where$a_1^L,, a_2^L, a_3^L, a_4^L, a_1^U, a_2^U, a_3^U, a_4^U, b_1^U, b_2^U, b_3^U$ *and* b_4^U are any real values, $0 \leq \hat{w}_{\tilde{A}^L} \leq \hat{w}_{\tilde{A}^U} \leq 1$ and $0 \leq \hat{w}_{\tilde{B}^L} \leq \hat{w}_{\tilde{B}^U} \leq 1$.

(2) Interval-valued fuzzy number subtraction $\ominus$ (Eq. (3):

$$\tilde{\tilde{A}} \ominus \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L;\ \hat{w}_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U;\ \hat{w}_{\tilde{A}^U}\right)\right] \ominus \left[\left(b_1^L, b_2^L, b_3^L, b_4^L;\ \hat{w}_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U;\ \hat{w}_{\tilde{B}^U}\right)\right] = [(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L ; min\left(\hat{w}_{\tilde{A}^L}, \hat{w}_{\tilde{B}^L}\right)) , (a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U) ; \min (\hat{w}_{\tilde{A}^U}, \hat{w}_{\tilde{B}^U}))] , \tag{3}$$

Where $a_1^L,, a_2^L, a_3^L, a_4^L, a_1^U, a_2^U, a_3^U, a_4^U, b_1^U, b_2^U, b_3^U$ *and* b_4^U are any real values, $0 \leq \hat{w}_{\tilde{A}^L} \leq \hat{w}_{\tilde{A}^U} \leq 1$ and $0 \leq \hat{w}_{\tilde{B}^L} \leq \hat{w}_{\tilde{B}^U} \leq 1$.

(1) Interval-valued fuzzy number multiplication $\otimes$ (Eq. (4)):

$$\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L;\ \hat{w}_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U;\ \hat{w}_{\tilde{A}^U}\right)\right] \otimes \left[\left(b_1^L, b_2^L, b_3^L, b_4^L;\ \hat{w}_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U;\ \hat{w}_{\tilde{B}^U}\right)\right] = [(a_1^L * b_1^L, a_2^L * b_2^L, a_3^L * b_3^L, a_4^L * b_4^L ; \min\left(\hat{w}_{\tilde{A}^L}, \hat{w}_{\tilde{B}^L}\right)), (a_1^U * b_1^U, a_2^U * b_2^U, a_3^U * b_3^U, a_4^U * b_4^U ;\min (\hat{w}_{\tilde{A}^U}, \hat{w}_{\tilde{B}^U}))] , \tag{4}$$

where$a_1^L,, a_2^L, a_3^L, a_4^L, a_1^U, a_2^U, a_3^U, a_4^U, b_1^U, b_2^U, b_3^U$ *and* b_4^U are any real values, $0 \leq \hat{w}_{\tilde{A}^L} \leq \hat{w}_{\tilde{A}^U} \leq 1$ and $0 \leq \hat{w}_{\tilde{B}^L} \leq \hat{w}_{\tilde{B}^U} \leq 1$.

(2) Interval-valued fuzzy number division $\oslash$ (Eq. (5)):

$$\tilde{\tilde{A}} \oslash \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L;\ \hat{w}_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U;\ \hat{w}_{\tilde{A}^U}\right)\right] \oslash \left[\left(b_1^L, b_2^L, b_3^L, b_4^L;\ \hat{w}_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U;\ \hat{w}_{\tilde{B}^U}\right)\right] = [(a_1^L / b_4^L, a_2^L / b_3^L, a_3^L / b_2^L, a_4^L / b_1^L ; min\left(\hat{w}_{\tilde{A}^L}, \hat{w}_{\tilde{B}^L}\right)) , (a_1^U / b_4^U, a_2^U / b_3^U, a_3^U / b_2^U, a_4^U / b_1^U) ; \min (\hat{w}_{\tilde{A}^U}, \hat{w}_{\tilde{B}^U}))] , \tag{5}$$

Where $a_1^L,, a_2^L, a_3^L, a_4^L, a_1^U, a_2^U, a_3^U, a_4^U, b_1^U, b_2^U, b_3^Uandb_4^U$ are all nonzero numbers or all nonzero negative real numbers, $0 \leq \hat{w}_{\tilde{A}^L} \leq \hat{w}_{\tilde{A}^U} \leq 1$ and $0 \leq \hat{w}_{\tilde{B}^L} \leq \hat{w}_{\tilde{B}^U} \leq 1$.

3.3. Linguistic terms

Suppose that the completion of an activity consists of uncertainty. As stated earlier the project experts can transform the uncertainty into a fuzzy number by assigning a membership function to this linguistic term, to indicate the relationship between the linguistic term and the fuzzy number (See Fig. 2). In this figure, the horizontal axis refers to the progress and is on a scale of 1. The summary of the transformation related to Fig. 2 is reported in Table 1. Clearly, the output of this transformation is an IVF-number.

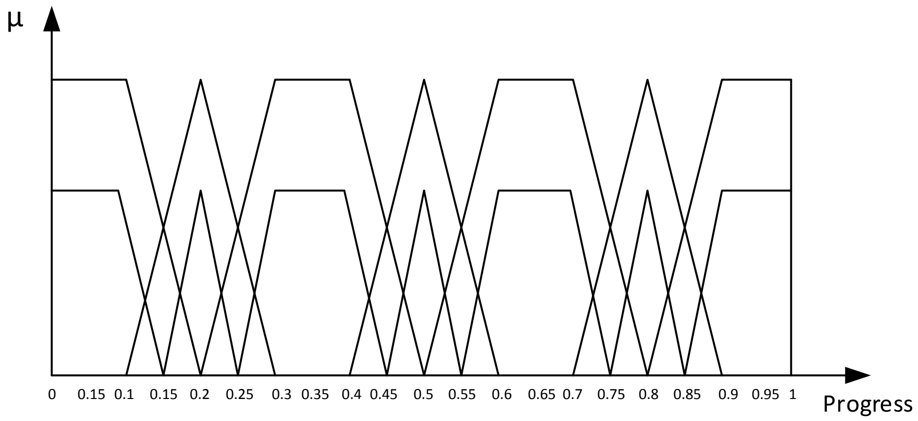


Fig. 2. An interval-valued fuzzy membership including trapezoidal and triangular interval-valued fuzzy numbers and the corresponding linguistic terms

Table 1
The assigned interval-valued fuzzy numbers to each linguistic term

Fuzzy Numbers	Linguistic Terms
[(0,0,0.1,0.15;0.6),(0,0,,0.1,0.2;1)]	Very Low
[(0.15,0.2,0.2,0.25;0.6),(0.1,0.2,0.2,0.3;1)]	Low
[(0.25,0.3,0.4,0.5;0.6),(0.2,0.3,0.4,0.5;1)]	Less than half
[(0.45,0.5,0.5,0.55;0.6),(0.4,0.5,0.5,0.6;1)]	Half
[(0.55,0.6,0.7,0.75;0.6),(0.5,0.6,0.7,0.8;1)]	More than half
[(0.75,0.8,0.8,0.85;0.6),(0.7,0.8,0.8,0.9;1)]	High
[(0.85,0.9,1,1;0.6),(0.8,0.9,1,1;1)]	Very high

For example, according to Fig. 2 and Table 1, the linguistic term "high" equals to the IVF-number [(0.75,0.8,0.8,0.85;0.6),(0.7,0.8,0.8,0.9;1)].

4. Proposed New Version of Earned Value Analysis with Interval-Valued Fuzzy Setting

In this section, proposed new version of the classical EVA is presented with IVF-settings for mega projects. At first, the interval-valued fuzzy-EV of activity i, $\tilde{\tilde{EV}}_i$should be derived (Eq. (6)) which itself is based on interval-valued fuzzy completion percent of the activity i(Eq. (7)).

$$\tilde{\tilde{EV}}_i = \tilde{\tilde{F}}_i * BAC_i = [(E_{1i}^L, E_{2i}^L, E_{3i}^L, E_{4i}^L; \widehat{w}_{\tilde{\tilde{EV}}^L}), (E_{1i}^U, E_{2i}^U, E_{3i}^U, E_{4i}^U; \widehat{w}_{\tilde{\tilde{EV}}^U})] \quad (6)$$

where

$$\tilde{\tilde{F}}_i = [(a_{1i}^L, a_{2i}^L, a_{3i}^L, a_{4i}^L; \widehat{w}_{\tilde{\tilde{F}}^L}), (a_{1i}^U, a_{2i}^U, a_{3i}^U, a_{4i}^U; \widehat{w}_{\tilde{\tilde{F}}^U})] \quad (7)$$

The BAC_i is the budget at completion of activity *i* and indicates the planned budget to complete the activity *i*. The total IVF-EVA in each measurement period, $\widetilde{\widetilde{EV}}$, is calculated by summing up all the $\widetilde{\widetilde{EV_i}}$ for *i*=1,2,…,*n* where *n* is the total number of the project's activities :

$$\widetilde{\widetilde{EV_i}}=\sum_{i=1}^{n}\widetilde{\widetilde{EV_i}}=[(\sum_{i=1}^{n}E_{1i}^{L},\sum_{i=1}^{n}E_{2i}^{L},\sum_{i=1}^{n}E_{3i}^{L},\sum_{i=1}^{n}E_{4i}^{L},),(\sum_{i=1}^{n}E_{1i}^{U},\sum_{i=1}^{n}E_{2i}^{U},\sum_{i=1}^{n}E_{3i}^{U},\sum_{i=1}^{n}E_{4i}^{U},)]=[(E_1^L,E_2^L,E_3^L,E_4^L;\hat{w}_{\widetilde{\widetilde{EV}}^L}),(E_1^U,E_2^U,E_3^U,E_4^U;\hat{w}_{\widetilde{\widetilde{EV}}^U})] \quad (8)$$

4.1. Interval-valued fuzzy performance indices

CPI and SPI are measurements to evaluate the cost performance and time progress of a project. CPI is a measure of the budgetary conformance of the actual cost of work performed, and SPI is a measure of the conformance of the actual progress to schedule.The schedule performance index is calculated as the earned value divided with the planned value, i.e.,$SPI=\frac{EV}{PV}$, where PV known also as the budgeted cost of work scheduled (BCWS) is scheduled to be used during the project. Eq.(9) shows the $\widetilde{\widetilde{SPI}}$ as below.

$$\widetilde{\widetilde{SPI}}=\frac{\widetilde{\widetilde{EV}}}{PV}=[\left(\frac{E_1^L}{PV},\frac{E_2^L}{PV},\frac{E_3^L}{PV},\frac{E_4^L}{PV};\hat{w}_{\widetilde{\widetilde{SPI}}^L}\right),\left(\frac{E_1^U}{PV},\frac{E_2^U}{PV},\frac{E_3^U}{PV},\frac{E_4^U}{PV};\hat{w}_{\widetilde{\widetilde{SPI}}^U}\right)] \quad (9)$$

The cost performance index is calculated as the EVA divided with actual cost (AC), i.e.,$PI=\frac{EV}{AC}$, where AC known also as the actual cost of work performed (ACWP), expresses the resources used to attain the actual work performed. The $\widetilde{\widetilde{CPI}}$ is as below in Eq. (10):

$$\widetilde{\widetilde{CPI}}=\frac{\widetilde{\widetilde{EV}}}{AC}=[\left(\frac{E_1^L}{AC},\frac{E_2^L}{AC},\frac{E_3^L}{AC},\frac{E_4^L}{AC};\hat{w}_{\widetilde{\widetilde{CPI}}^L}\right),\left(\frac{E_1^U}{AC},\frac{E_2^U}{AC},\frac{E_3^U}{AC},\frac{E_4^U}{AC};\hat{w}_{\widetilde{\widetilde{CPI}}^U}\right)] \quad (10)$$

The critical ratio (CR), called as schedule cost index (SCI), is a multiplication of the SPI by CPI and indicates the project health and is calculated using Eq. (11):

$$\widetilde{\widetilde{CR}}=\widetilde{\widetilde{SCI}}=\widetilde{\widetilde{SPI}}*\widetilde{\widetilde{CPI}}=[\left(\frac{{E_1^L}^2}{PV*AC},\frac{{E_2^L}^2}{PV*AC},\frac{{E_3^L}^2}{PV*AC},\frac{{E_4^L}^2}{PV*AC};\hat{w}_{\widetilde{\widetilde{CR}}^L}\right),\left(\frac{{E_1^U}^2}{PV*AC},\frac{{E_2^U}^2}{PV*AC},\frac{{E_3^U}^2}{PV*AC},\frac{{E_4^U}^2}{PV*AC};\hat{w}_{\widetilde{\widetilde{CR}}^U}\right)] \quad (11)$$

4.2. Interval-valued fuzzy cost estimates at completion

EVA can also be used to forecast cost and time at project completion. For both cost and duration forecasting, several formulas have been introduced in the related literature. Among several formulas available to calculate cost estimate at completion (EAC), a common formula presumes the future trend of the project cost performance remains untouched. In this formula EAC is calculated by dividing the budget at completion (BAC) by CPI. In other words, this formula assumes that the CPI would be fixed during the rest of the project. The interval-valued fuzzy-EAC in Eq. (12) is as below:

$$\widetilde{\widetilde{EAC}}=\frac{BAC}{\widetilde{\widetilde{CPI}}}=\frac{BAC}{[\left(\frac{E_1^L}{AC},\frac{E_2^L}{AC},\frac{E_3^L}{AC},\frac{E_4^L}{AC}\right),\left(\frac{E_1^U}{AC},\frac{E_2^U}{AC},\frac{E_3^U}{AC},\frac{E_4^U}{AC}\right)]}=[(\frac{BAC*AC}{E_4^L},\frac{BAC*AC}{E_3^L},\frac{BAC*AC}{E_2^L},\frac{BAC*AC}{E_1^L};\hat{w}_{\widetilde{\widetilde{EAC}}^L}),(\frac{BAC*AC}{E_4^U},\frac{BAC*AC}{E_3^U},\frac{BAC*AC}{E_2^U},\frac{BAC*AC}{E_1^U};\hat{w}_{\widetilde{\widetilde{EAC}}^U})] \quad (12)$$

EAC could also be affected both current cost performance and current schedule performance indices. Another formula for EAC based on this assumption in Eq.(13) is shown as follows:

$$\widetilde{\widetilde{EAC}} = AC + [\frac{BAC-\widetilde{\widetilde{EV}}}{\widetilde{\widetilde{SCI}}}] = AC + [\frac{BAC-[(E_1^L,E_2^L,E_3^L,E_4^L),(E_1^U,E_2^U,E_3^U,E_4^U)]}{[\left(\frac{{E_1^L}^2}{PV*AC},\frac{{E_2^L}^2}{PV*AC},\frac{{E_3^L}^2}{PV*AC},\frac{{E_4^L}^2}{PV*AC}\right),\left(\frac{{E_1^U}^2}{PV*AC},\frac{{E_2^U}^2}{PV*AC},\frac{{E_3^U}^2}{PV*AC},\frac{{E_4^U}^2}{PV*AC}\right)]} =$$

$$AC + \left[\left(\frac{BAC-E_4^L}{\frac{{E_4^L}^2}{PV*AC}},\frac{BAC-E_3^L}{\frac{{E_3^L}^2}{PV*AC}},\frac{BAC-E_2^L}{\frac{{E_2^L}^2}{PV*AC}},\frac{BAC-E_1^L}{\frac{{E_1^L}^2}{PV*AC}};\widehat{w}_{\widetilde{\widetilde{EAC}}^L}\right),\left(\frac{BAC-E_4^U}{\frac{{E_4^U}^2}{PV*AC}},\frac{BAC-E_3^U}{\frac{{E_3^U}^2}{PV*AC}},\frac{BAC-E_2^U}{\frac{{E_2^U}^2}{PV*AC}},\frac{BAC-E_1^U}{\frac{{E_1^U}^2}{PV*AC}};\widehat{w}_{\widetilde{\widetilde{EAC}}^U}\right)\right] \quad (13)$$

4.3. Interval-valued fuzzy time estimates at completion

Despite being expanded to control both time and cost, most of the researches have been concentrated on the cost aspects (e.g., Fleming & Koppelman, 2005). Thus, the EV technique has not been widely used to estimate the total time at completion. In response to this problem, Lipke (2004) introduced the earned schedule (ES) indicators and later developed by Henderson (2004). The ES indicators are time-based indicators instead of cost-based indicators. Since time delays are measured versus time, the ES indicators are reliable during the whole time horizon of the project as the adopted formula for schedule performance does not converge to 1.

In this paper, the ES technique has been used to develop the IVF-based time estimate at completion. Since the EAC represents the estimate of the completion cost, to avoid confusion in notation. We follow $EAC_t$to reflect the time estimate at completion.According to Lipke (2003), the ES is time equivalent of the EV which is resulted by projecting the EV on the baseline. Hence, it measures schedule performance using time. Eq.(14) illustrates mathematics of the ES:

$$ES = N + (\frac{EV - PV_N}{PV_{N+1} - PV_N} \quad (14)$$

Where N is the longest time interval in which the PV_N is less than EV, the PV_N is the planned value at time N and PV_{N+1} is the planned value at the next time interval, i.e., time N+1 (Vandervoorde and Vanhouck, 2005).

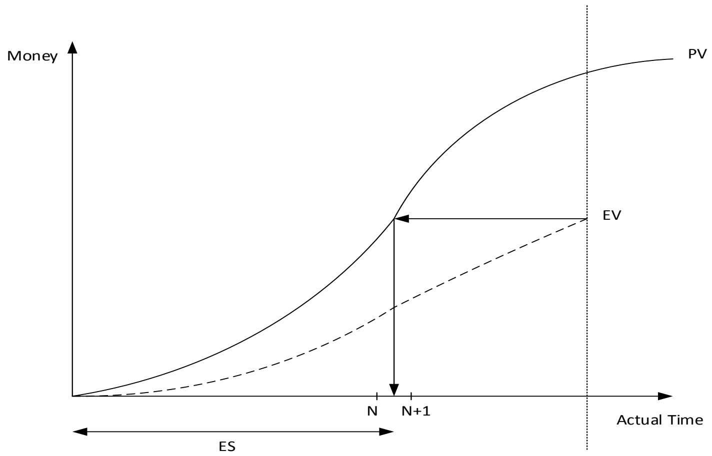


Fig. 3. The ES versus the EV

The IVF-ES could simply be set by Eq. (16).

$$ES_i = N_i + \left(\frac{E_i - PV_N}{PV_{N+1} - PV_N}\right) \tag{15}$$

$$\widetilde{\widetilde{ES}} = [\left(ES_1^L, ES_2^L, ES_3^L, ES_4^L; \hat{w}_{\widetilde{\widetilde{ES}}^L}\right), \left(ES_1^U, ES_2^U, ES_3^U, ES_4^U; \hat{w}_{\widetilde{\widetilde{ES}}^U}\right)] \tag{16}$$

In contrast to the SPI, SPI_t is illustrated in time units and is the ratio of ES to the actual duration (AD), i.e. $\widetilde{\widetilde{SPI_t}} = \frac{\widetilde{\widetilde{ES}}}{AD}$. Eq. (17) represents as below:

$$\widetilde{\widetilde{SPI_t}} = \frac{\widetilde{\widetilde{ES}}}{AD} = [\left(\frac{ES_1^L}{AD}, \frac{ES_2^L}{AD}, \frac{ES_3^L}{AD}, \frac{ES_4^L}{AD}; \hat{w}_{\widetilde{\widetilde{SPI_t}}^L}\right), \left(\frac{ES_1^U}{AD}, \frac{ES_2^U}{AD}, \frac{ES_3^U}{AD}, \frac{ES_4^U}{AD}; \hat{w}_{\widetilde{\widetilde{SPI_t}}^U}\right)] \tag{17}$$

The generic ES-based equation to estimate the time at completion of project is:

$$EAC_t = AD + (\frac{PD - ES}{PF}) \tag{18}$$

where PD is the planned duration to complete the project and PF is the performance factor which relies on the project status. Generally, the following three cases are considered for the PF:

- PF=1 : The duration of the remained activities is as planned.

$$\widetilde{\widetilde{EAC_t}} = AD + \left(PD - \widetilde{\widetilde{ES}}\right) = [\left(AD + PD - ES_4^L, AD + PD - ES_3^L, AD + PD - ES_2^L, AD + PD - ES_1^L; \hat{w}_{\widetilde{\widetilde{EAC_t}}^L}\right), \left(AD + PD - ES_4^U, AD + PD - ES_3^U, AD + PD - ES_2^U, AD + PD - ES_1^U; \hat{w}_{\widetilde{\widetilde{EAC_t}}^U}\right)] \tag{19}$$

- PF=$\widetilde{\widetilde{SPI_t}}$: The duration of remained activities changes with the current $\widetilde{\widetilde{SPI_t}}$ trend.

$$\widetilde{\widetilde{EAC_t}} = AD + (\frac{PD - \widetilde{\widetilde{ES}}}{\widetilde{\widetilde{SPI_t}}}) = AD + [\frac{[(PD-ES_4^L, PD-ES_3^L, PD-ES_2^L, PD-ES_1^L), (PD-ES_4^U, PD-ES_3^U, PD-ES_2^U, PD-ES_1^U)}{\widetilde{\widetilde{SPI_t}}} = AD + [\left(\frac{PD-ES_4^L}{\frac{ES_4^L}{AD}}, \frac{PD-ES_3^L}{\frac{ES_3^L}{AD}}, \frac{PD-ES_2^L}{\frac{ES_2^L}{AD}}, \frac{PD-ES_1^L}{\frac{ES_1^L}{AD}}; \hat{w}_{\widetilde{\widetilde{EAC_t}}^L}\right), \left(\frac{PD-ES_4^U}{\frac{ES_4^U}{AD}}, \frac{PD-ES_3^U}{\frac{ES_3^U}{AD}}, \frac{PD-ES_2^U}{\frac{ES_2^U}{AD}}, \frac{PD-ES_1^U}{\frac{ES_1^U}{AD}}; \hat{w}_{\widetilde{\widetilde{EAC_t}}^U} \right. \tag{20}$$

- PF=$\widetilde{\widetilde{SCI_t}}$: The duration of remained activities changes with the $\widetilde{\widetilde{SCI_t}}$ trend, where $\widetilde{\widetilde{SCI_t}} = \widetilde{\widetilde{SPI_t}} * \widetilde{\widetilde{CPI}}$

$$\widetilde{\widetilde{EAC_t}} = AD + \left(\frac{PD - \widetilde{\widetilde{ES}}}{\widetilde{\widetilde{SCI_t}}}\right) = AD + [\left(\frac{PD - ES_4^L}{\frac{ES_4^L * E_4^L}{AD*AC}}, \frac{PD - ES_3^L}{\frac{ES_3^L * E_3^L}{AD*AC}} \frac{PD - ES_2^L}{\frac{ES_2^L * E_2^L}{AD*AC}} \frac{PD - ES_1^L}{\frac{ES_1^L * E_1^L}{AD*AC}}; \hat{w}_{\widetilde{\widetilde{SCI_t}}^L}\right), \left(\frac{PD - ES_4^U}{\frac{ES_4^U * E_4^U}{AD*AC}}, \frac{PD - ES_3^U}{\frac{ES_3^U * E_3^U}{AD*AC}} \frac{PD - ES_2^U}{\frac{ES_2^U * E_2^U}{AD*AC}} \frac{PD - ES}{\frac{ES_1^U * E_1^U}{AD*AC}} \right. \tag{21}$$

4.4. Interpretation of interval-valued fuzzy performance indices

After expanding the interval-valued fuzzy-based EV indices, we should interpret them to have a conclusion concerning the mega project progress and its status. Similar to the traditional EV indices where a comparison is made against the value 1 which represents the EV against planned value and actual cost, the same story 0goes here, i.e., the new interval-valued fuzzy indices must be compared against value 1. But as the new indices are interval-valued fuzzy numbers, we should transform them to IVF-numbers to be able to compare them using the methods proposed for comparing the IVF-numbers. Eq. (22) shows how to transform the IVF-number $\tilde{A}^*$.

$$\tilde{A}^* = (a_1^*, a_2^*, a_3^*, a_4^*; w_{\tilde{A}^*}) = (\frac{a_1^U + a_1^L}{2}, \frac{a_2^U + a_2^L}{2}, \frac{a_3^U + a_3^L}{2}, \frac{a_4^U + a_4^L}{2}; \frac{w_{\tilde{\tilde{A}}^U} + w_{\tilde{\tilde{A}}^L}}{2}) \tag{22}$$

Different methods introduced for comparing fuzzy numbers in the literature (See Bortolan and Degani, 1985; Dubois and Prade, 1980), a well-known fuzzy ranking method proposed by Dubois and Prade (1980) measures the degree of possibility by which a fuzzy number is either greater or less than another fuzzy number. Regarding to Dubois and Prade (1980), $T(\tilde{A})$ is the degree of possibility of fuzzy number $\tilde{A}$, thus $T(\tilde{A})$=$\mu_{\tilde{A}}(x)$ where $x \in \tilde{A}$. Given two fuzzy numbers $\tilde{A}, \tilde{B}$, the degree of possibility that $\tilde{A} \geq \tilde{B}$ is

$$T(\tilde{A} \geq \tilde{B}) = Sup_{x \geq y} min\{\mu_{\tilde{A}}(x) , \mu_{\tilde{B}}(x)\}. \tag{23}$$

The proposed model conducts to deterministic conclusions almost in all cases, however in several cases the results are not deterministic which might be of interest as it much closer to the nature of project and the proposed new uncertain model in this paper. To apply Eq. (23) to $\widetilde{SPI} = \frac{\widetilde{EV}}{PV} = [\frac{E_1}{PV}, \frac{E_2}{PV}, \frac{E_3}{PV}, \frac{E_4}{PV}]$ and $\widetilde{CPI} = \frac{\widetilde{EV}}{AC} = [\frac{E_1}{AC}, \frac{E_2}{AC}, \frac{E_3}{AC}, \frac{E_4}{AC}]$; however, the comparison is made against 1, thus

$$T(\widetilde{SPI} \geq 1) = Sup_{x \geq 1} \min\{\mu_{\widetilde{SPI}}(x) = 1\} = Sup_{x \geq 1}\{\mu_{\widetilde{SPI}}(x)\} \tag{24}$$
$$T(\widetilde{CPI} \geq 1) = Sup_{x \geq 1} \min\{\mu_{\widetilde{CPI}}(x) = 1\} = Sup_{x \geq 1}\{\mu_{\widetilde{CPI}}(x)\} \tag{25}$$

Supposing trapezoidal fuzzy numbers, thus both $T(\widetilde{SPI} \geq 1)$and $T(\widetilde{CPI} \geq 1)$ concludes in five scenarios as Tables 2 and 3 show, respectively. In these tables, the vertical line depicts the position of value 1, as we would like to compare $\widetilde{SPI}$ and $\widetilde{CPI}$ to this value. Regarding to the tables, the degree of possibility of $\widetilde{CPI} \geq 1$and $\widetilde{SPI} \geq 1$, i.e. $T(\widetilde{SPI} \geq 1)$ and $T(\widetilde{CPI} \geq 1)$ are between 0 and 1.

The proximity of $T(\widetilde{CPI} \geq 1)$ to 1, for example, explains the project is ahead of budget, and its proximity to 0 shows, the project is behind the budget. As mentioned earlier, this comparison does not always give crisp decisions, which is more appropriate and consistent with the basis of the proposed IVF-EVA model.

For scenarios 2 and 4 in Tables 2 and 3, the values of $T(\widetilde{SPI} \geq 1)$ and $T(\widetilde{CPI} \geq 1)$ are depicted in the graphical descriptions by black bold lines (as a part of original red line). Hence, for example in scenario 2 of Table 2, the length of the black bold line inside the trapezoidal is equal to $T(\widetilde{CPI} \geq 1) = Sup_{x \geq 1}\{\mu_{\widetilde{CPI}}(x)\} = \frac{d-1}{c-1}$.

Note that regarding to the fuzzy principles we cannot draw conclusions for $T(\widetilde{CPI} \leq 1)$ based on $T(\widetilde{CPI} \geq 1)$, thus we calculated $T(\widetilde{CPI} \geq 1)$ separately (see Table 2). This rule applies also to$T(\widetilde{SPI} \leq 1)$(See Table 3).

5. Application Example

In this section, an application example from the recent literature (Naeini et al., 2013)has been brought for explaining the basis of the interval-valued fuzzy-based EV calculations. This example refers to a medical research project. Hospital sampling is the case that has been illustrated in this section and has consisted of the six activities ‘filling questionnaire’, ‘checking-up people’, ‘getting sample’, ‘testing sample’, ‘ analyzing sample’ and ‘refining sample’. In this project, it was needed to collect 600 hospital samples. These six activities lay over a period of 12 weeks which Table 4 indicates the planned values (PV) and actual costs (AC) up to data date, i.e. week 6. The information about activity progress and activity budget at completion (BAC) are brought in Table 5. As Table 7 indicates, the progress of all activities is stated using linguistic terms. These linguistic terms are transformed into IVF-numbers ($\widetilde{F}_i$) using Table 1, and they have brought in Table 6.

Table 2
The degree of possibility of $\widetilde{CPI} \geq 1$ and $\widetilde{CPI} \leq 1$

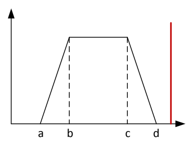
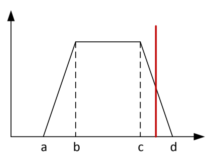
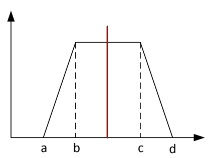
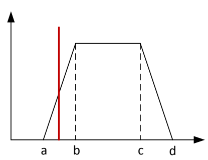
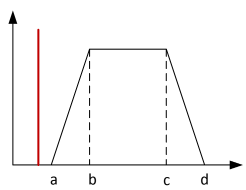
No. of scenarios	State of $\widetilde{CPI}$ against 1	Degree of possibility of $\widetilde{CPI} \geq 1$	Degree of possibility of $\widetilde{CPI} \leq 1$	Graphical Description	Decision making
1	$d < 1$	$T(\widetilde{CPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = 0$	$T(\widetilde{CPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = 1$		Behind the budget
2	$c < 1 < d$	$T(\widetilde{CPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = \frac{d-1}{d-c}$	$T(\widetilde{CPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = 1$		Approximately behind the budget
3	$b < 1 < c$	$T(\widetilde{CPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = 1$	$T(\widetilde{CPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = 1$		On the budget
4	$a < 1 < b$	$T(\widetilde{CPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = 1$	$T(\widetilde{CPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = \frac{1-a}{b-a}$		Approximately ahead of budget
5	$a > 1$	$T(\widetilde{CPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = 1$	$T(\widetilde{CPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{CPI}}(x)\} = 0$		Ahead of budget

Table 3
The degree of possibility of $\widetilde{SPI} \geq 1$ and $\widetilde{SPI} \leq 1$

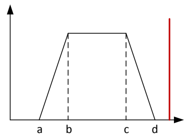
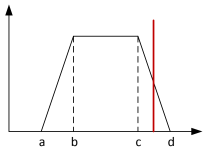
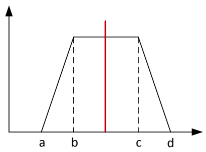
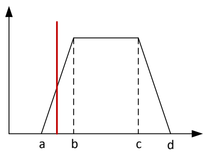
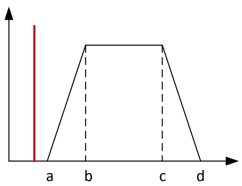
No. of scenarios	State of $\widetilde{SPI}$ against 1	Degree of possibility of $\widetilde{CPI} \geq 1$	Degree of possibility of $\widetilde{SPI} \leq 1$	Graphical Description	Decision making
1	$d < 1$	$T(\widetilde{SPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = 0$	$T(\widetilde{SPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = 1$		Behind the schedule
2	$c < 1 < d$	$T(\widetilde{SPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = \frac{d-1}{d-c}$	$T(\widetilde{SPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = 1$		Approximately behind the schedule
3	$b < 1 < c$	$T(\widetilde{SPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = 1$	$T(\widetilde{SPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = 1$		On the schedule
4	$a < 1 < b$	$T(\widetilde{SPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = 1$	$T(\widetilde{SPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = \frac{1-a}{b-a}$		Approximately ahead of schedule
5	$a > 1$	$T(\widetilde{SPI} \geq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = 1$	$T(\widetilde{SPI} \leq 1) = Sup_{x\geq 1}\{\mu_{\widetilde{SPI}}(x)\} = 0$		Ahead of schedule

Table 4
The PV and the AC of the example

Week	1	2	3	4	5	6	7	8	9	10	11	12
PV	100	400	750	1200	1700	2300	2950	3600	4150	4650	4850	5000
AC	200	500	1000	1500	2000	2800	-	-	-	-	-	-

Table 5
The information of work package of the application example

Activity	BAC($)	Progress
1. Filling questionnaire	800	Very high
2. Checking-up people	1000	More than half
3. Getting Sample	500	More than half
4. Testing sample	1200	Less than half
5. Analyzing sample	900	Very low
6. Refining sample	600	Not started

Table 6
The percent complete and the EV of activities

Activity(i)	BAC($)	Progress	$\widetilde{\widetilde{F_i}}$	$\widetilde{\widetilde{EV_i}}$
1	800	Very high	[(0.85,0.9,1,1;0.6),(0.8,0.9,1,1;1)]	[(680,720,800,800;0.6),(640,720,800,800;1)]
2	1000	More than half	[(0.55,0.6,0.7,0.75;0.6),(0.5,0.6,0.7,0.8;1)]	[(550,600,700,750;0.6),(500,600,700,800;1)]
3	500	More than half	[(0.55,0.6,0.7,0.75;0.6),(0.5,0.6,0.7,0.8;1)]	[(275,300,350,375;0.6),(250,300,350,400;1)]
4	1200	Less than half	[(0.25,0.3,0.4,0.45;0.6),(0.2,0.3,0.4,0.5;1)]	[(300,360,480,540;0.6),(240,360,480,600;1)]
5	900	Very low	[(0,0,0.1,0.15;0.6),(0,0,0.1,0.2;1)]	[(0,0,90,135;0.6),(0,0,90,180;1)]
6	600	Not started	[(0,0,0,0;0.6),(0,0,0,0;1)]	[(0,0,0,0;0.6),(0,0,0,0;1)]

In accordance with the BAC of each activity, the $\widetilde{\widetilde{EV}}$ of each activity is calculated using Eq.(3). Using Eq.(5), we can find the total EV of the work package, i.e. the total EV for all of the four activities:

$$\widetilde{\widetilde{EV}} = \sum_{i=1}^{8} \widetilde{\widetilde{EV_i}} = [(1805,1980,2420,2600;0.6),(1630,1980,2420,2780;1)]$$

As Table 6shows, the total PV and the total AC for the work package at the data date, i.e., week 6 are 2300$ and 2800$, respectively. The SPI and CPI of this work package are interpreted based on Eq. (10) as follows:

$$\widetilde{\widetilde{CPI}} = \frac{\widetilde{\widetilde{EV}}}{AC} = [(0.64,0.71,0.86,0.93;0.6),(0.58,0.71,0.86,0.99;1)]$$

$$\widetilde{CPI^*} = [(0.61,0.71,0.86,0.96;0.8)]$$

According to Table 2 (see scenario 1), $T(\widetilde{CPI^*} > 1) = 0$and $T(\widetilde{CPI^*} \le 1) = 1$. Thus, the work package is behind the budget.

$$\widetilde{\widetilde{SPI}} = \frac{\widetilde{\widetilde{EV}}}{PV} = \begin{bmatrix}(0.78,0.86,1.05,1.13;0.6),\\(0.71,0.86,1.05,1.21;1)\end{bmatrix}$$

$$\widetilde{SPI^*} = [(0.75,0.86,1.05,1.17;0.8)]$$

Scenario 3 in Table 3 implies that $T(\widetilde{SPI^*} \le 1) = 1$ and $T(\widetilde{CPI^*} > 1) = 1$, so the work package is on the schedule. These results illustrate that this work package is behind the budget and on the schedule. Now, we will estimate the completion cost of the work package. Given the trend affecting the CPI is kept fixed during the rest of the project, the completion cost of the work package is estimated as follows. Regarding to Table 7, the total BAC for all activities is 5000$.

$$\widetilde{\widetilde{EAC}} = \frac{BAC}{\widetilde{\widetilde{CPI}}} = \begin{bmatrix}(5035.97,5785.12,7070.71,8588.96;0.6),\\(5384.61,5785.12,7070.71,7756.23;1)\end{bmatrix}$$

Table 7
The summary of $\widetilde{\widetilde{EAC_t}}$ calculations

I	E_i	N_i	$E_i - PV_{N_i}$	$PV_{N_i+1} - PV_{N_i}$	$ES_i = N_i + \frac{E_i - PV_{N_i}}{PV_{N_i+1} - PV_{N_i}}$
1	1805	5	105	600	5.175
2	1980	5	280	600	5.47
3	2420	6	120	650	6.18
4	2600	6	300	650	6.46
5	1630	4	430	500	4086
6	1980	5	280	600	5.47
7	2420	6	120	650	6.18
8	2780	6	480	650	6.46

To calculate the $\widetilde{\widetilde{ES}}$, each of the eight numbers of the trapezoidal $\widetilde{\widetilde{EV}}$ is projected on the baseline and forms a member of the trapezoidal interval-valued fuzzy number $\widetilde{\widetilde{ES}}$ (Eq.(12)). For example, the calculations behind the ES_1 are:

$N_1 = 5$as$PV_5 < E_1 = 1805 < PV_6$ thus

$$ES_1 = N_1 + \left(\frac{E_1 - PV_5}{PV_6 - PV_5}\right) = 5 + \left(\frac{1805 - 1700}{2300 - 1700}\right) = 5.175$$

Following these calculations,

$\widetilde{\widetilde{ES}} = [(5.17,5.47,6.18,6.46; 0.6), (4.86,5.47,6.18,6.46; 1)]$

, Table 7 also indicates how $\widetilde{\widetilde{EAC_t}}$should be calculated in the developed model.

To obtain the $\widetilde{\widetilde{SPI_t}}$ by Eq. (13) and the $\widetilde{\widetilde{EAC_t}}$ by Eq.(17) given PF=$\widetilde{\widetilde{SPI_t}}$) we have:

$$\widetilde{\widetilde{SPI_t}} = \frac{\widetilde{\widetilde{ES}}}{AD} = \begin{bmatrix}(0.86,0.91,1.03,1.08; 0.6),\\ (0.81,0.91,1.03,1.08; 1)\end{bmatrix}$$

$$\widetilde{\widetilde{EAC_t}} = \begin{bmatrix}(11.13,11.65,13.18,13.94; 0.6),\\ (11.13,11.65,13.18,14.18; 1)\end{bmatrix}$$

Here, the comparison between the presented IVF model and the classical fuzzy model that has been brought in Naeni et. al. (2013) is reported in Tables 8 and 9. In these tables, the maximum amounts of SPI and CPI are compared against the baseline amount, i.e. 1 and also against each other to determine the differences between the results of two models.

Table 8
Comparative analysis of SPI

	Max(SPI)	SPI Vs. 1
Proposed IVF model	1.17	$SPI > 1$
Classical fuzzy model by Naeni et. al. (2013)	1.21	$SPI > 1$

Table 9
Comparative analysis of CPI

	Max(CPI)	CPI Vs. 1
Proposed IVF model	0.96	$SPI < 1$
Classical fuzzy model by Naeni et. al. (2013)	0.99	$SPI < 1$

According to the results that have been gotten from the presented model and comparing them to the classical fuzzy method by Naeni et. al. (2013), it can be concluded that the results from both models are coincide and close to each other. Considering the advantages of IVFSs theory over classical fuzzy sets; for example, the fact that IVFS is an increasingly popular extension of fuzzy set theory where traditional [0,1] valued membership degrees are replaced by intervals in [0,1], in which we can approximate the (partially unknown) exact degrees. It not only handles vagueness but also addresses a feature of uncertainty (lack of information) intuitively by IVFSs. It shows that the results by model based on IVFSs are more reliable and accurate than the classical fuzzy sets.

6. Conclusions

Evaluating the EV is one of the primary stages in executing the EVA. The EV is based on measuring the performance of mega projects. To measure the performance a couple of different techniques have been developed. The percent complete is the simplest and the most applicable technique to measure the EV; however, it has the disadvantages of using subjective judgments when describing the percent completed of work. In the other view, the interval-valued fuzzy (IVF)-estimates of both completion cost and time can help project managers to estimate the future position of the mega project in a more trustable way. Integrating the fuzzy principles with EV formulas does not have the limitations of the previous deterministic EVA models and it can model the mega project in close proximity with reality. In this paper, we have proposedthe linguistic terms and new interval-valued fuzzy approach for evaluating the EVA, namely, IVF-EVA. We have described comprehensively how the IVF-EV indices and estimates can be expanded and clarified. In the clarification of the IVF- indices, we have benefited from measuring the degree of possibility of an IVF-number in taking various values. As it mentioned before, SPI and CPI are two important indices that help the project managers to measure and evaluate the schedule and cost of a project. Also, IVFSs have more advantages rather to classical fuzzy sets, like being more flexible in confronting uncertainty conditions. Thus, $\tilde{\tilde{SPI}}$ and $\tilde{\tilde{CPI}}$ in this paper show that the project is behind the budget and on the schedule under uncertain conditions. Hence, there should be some correction decisions from the project team to make it on the budget. Also, computational results of the presented model have been compared with classical fuzzy method from the recent literature. It was concluded that the results from both models are coincide and close to each other. Real-life applications of the proposed IVF-EVA have persuaded the researchers to continue the study on the application of the extended fuzzy sets theory in the EVA calculations and interpretations. Evaluating EAV indices with intuitionistic fuzzy sets (IFSs) can be taken into consideration as future research.

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