Development of a Novel Lot-sizing Model with Variable Lead Time in Supply Chain Environment

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Abstract

Supply chain management (SCM) addresses the management of materials and information across the entire chain from suppliers to producers, distributors, retailers, and customers. The theory of supply chain management suggests that lead time reduction is the pioneer of using market mediation to reduce transaction uncertainty in the chain, which can be conceptualized as the primary goal of supply chain management. In the past few decades, scholars have had to place attention on the impact of inventory on SCM. This paper is related to the development of a lot-sizing model for a single-component multiple-delivery system with variable demand and lead time of a multinational transformer company. Two models were developed. For the first model, distribution of demand is considered as normal, distribution of procurement lead time is exponential, and the quantity is coming in a single lot. For the second model, distribution of demand is normal, and procurement and administrative delay lead time is exponential, and the quantity is coming in a single lot.
Modification of the first model has been incorporated by taking the effect of multiple deliveries on the models and correcting the Re-order point as obtained from the previous models. The results and analysis by the second model have been done for different parametric conditions. The effect of multiple deliveries is also taken into account. The optimum re-order point and economic ordering quantity with various different inputs have been discussed.

Keywords: Supply chain management, Lot size, Economic order quality, Lead time, Re-order point.

1. Introduction

A lot size is a measure, or quantity addition, acceptable to or specified by a party offering to buy or sell it. It is also used as an alternative term for lot quantity of goods purchased or produced in expectation of the use or sale in the future. Four types of lot-sizing techniques are available: (1) the EOQ, followed by some assumptions for reducing inventory in smaller lots; (2) dynamic lot-sizing, considering the problem of determining production lot sizes when demand is deterministic, but varies with the time; (3) fixed order quantity, a policy is to produce a fixed amount each time by performing a setup; (4) part-period balancing, an idea to balance the inventory-carrying and setup costs.
Traditionally, any manufacturing or service organization performs purchasing, producing, and marketing activities independently, so that it is difficult to make an optimal plan for the supply chain. Research pieces in supply chain management have pre-focused on three major issues: (1) complexity in information flow; (2) mode in management; (3) planning of operation management for processing. In this research, this study puts emphasis on the cost factors, and the effect of the cost factor on supply chain is studied.
It is mentioned here that effective lot size is expected to optimize the supply chain and unwanted cash flow and reduce the possibility of occurrence on inventory shortage caused by variable orders.
A reciprocal cause and effect relationship exists between production planning and control and inventory levels at various stages during processing, and this affects the system.
Eltogral et al. (2007) introduced the complete solution of the problem in an explicit and extended manner. The authors incorporated transportation cost explicitly into the model and developed optimal solution procedures for solving the integrated models. In this paper, they have also developed two new models that integrate the transportation cost explicitly in the single-vendor single-buyer problem. The transportation cost is considered to be in an all-unit-discount format for the first model. The option of over declaring a shipment to exploit the transportation unit cost structure is explored in the second model. The objective in both models is to view the system

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be calculated with various parameters. Results from the model can be analyzed to investigate the sensitivity of the system in different parametric conditions. Ganeshan (1999) considered low-level inventory to start with and optimized the cost parameter.

In this present paper, a case study has been considered for a power transformer manufacturing unit located in India. The case of a single-item multiple-delivery system with variable demand and lead time has been investigated. In this case, the demand is distributed normally, lead time is distributed exponentially, and the quantity arrives in sub-lots. For manufacturing different ranges of products, the company has to purchase and stock the inputs required for production. Some items may bring into price advantage for larger purchases as suggested by Seo et al. (2002). When the input materials have longer varying lead time, the company has to keep a large amount of inventories as suggested by Tee and Rossetti (2002). In certain cases where input materials have shorter and constant lead time, the company may keep a lower level of inventories. The company has a centralized purchasing procedure for A and some of the B items of its A.B.C. list. Some B items and all C items are purchased independently by the different units of the company.

At the beginning of every month, in this case study, with the production plan in hand, each production unit starts issuing materials and components for A items from central stores and the remaining B and C items from the stores. A summary of issuing materials is recorded according to the types of materials that have been issued. As the consumption date of the items from stores was not available, the monthly production data of the transformers is collected for one year for one of the production unit of the company for analysis. Though the company has two production units producing the power transformers, the case investigated here is limited only for one production unit. Here, it is assumed that the production rate is equal to the demand rate for A items.

The budgeted production schedule of the product (in this case study, the products are power transformers of 50 MvA to 350 MvA) for a year was available and is shown in the Table 1, and the variation is shown in Fig. 2.

Table 1
Budgeted Production Plan for Transformers

Month	Monthly production (No.)
Jan	2
Feb	12
March	7
April	5
May	5
June	2
July	5
Aug	20
Sept	13
Oct	12
Nov	25
Dec	7
Total	115

Considering the discussion with the Head of Purchasing Department, the authors have taken carrying cost of inventory equal to 30% per year. It is the same with the company's norm.

The Company has ordered 1200 purchase orders for transformer components. The accounting department has taken costs incurred for these 646 orders to be Rs. 16818000/; so, cost per order is Rs. 16818000/1200 = Rs. 14013/- approximately

The ordering cost is divided into the following parts, as shown in the Table 2.

The unit cost of one A item (for this case study, it is "Cold Rolled Grain Oriented" steel) supplied by the vendors in a year is given in the Table 3 followed by Fig. 3.

For the CRGO, the frequency diagram for usage is shown in Fig. 4, and χ^2 test has been carried out in Table 4. χ^2 value from the Table 4 with degrees of freedom 3 and at 5% level is 7.815. So, the fit is not very good, because the materials are supplied according to the received order of the transformers.

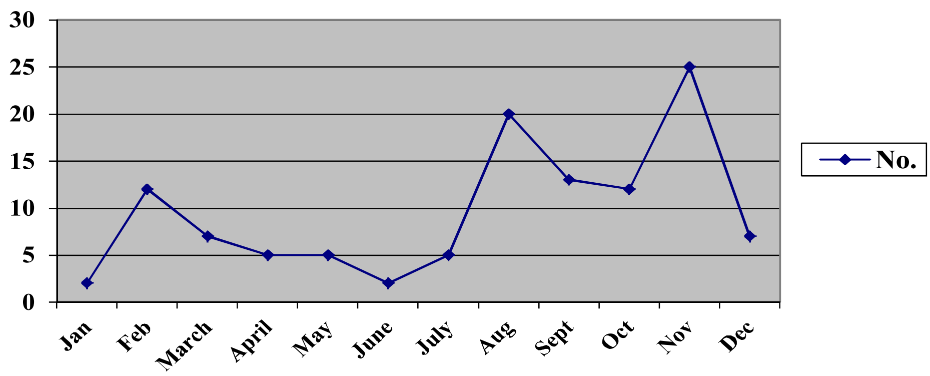

Fig. 2. Monthly production plan for transformers

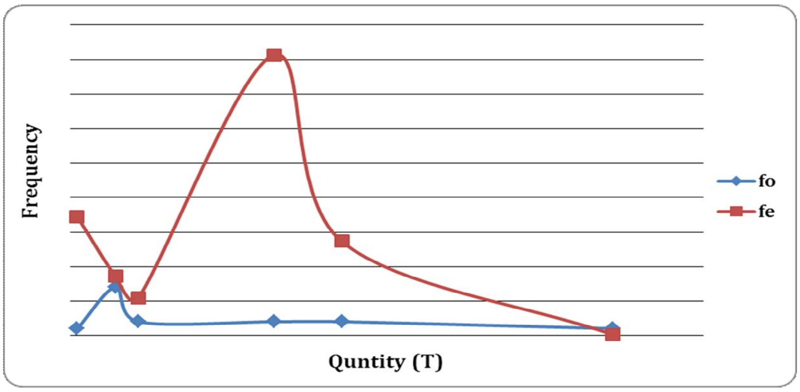

Fig. 4. Frequency distribution of quantity (T) CRGO

Received and used Quantities are shown in Table 5 followed by Fig. 5.

Table 5
Quantity of CRGO received.

Month	Quantity received (T)	Balanced Quantity (T)	Quantity used (T)	No.
Jan	69	69	8	1
		61	8	1
Feb	291	344	63	1
		281	64	1
		217	200	10
March	137	154	100	5
March		54	40	2
April	100	114	100	5
May	150	164	100	5
June	100	164	156	2
July	100	108	90	5
August	600	618	100	4
		518	286	13
		232	28	2
		204	19	1
Sep	450	635	455	7
		180	108	6
Oct	600	672	218	4
		454	157	2
		297	88	3
		209	159	3
Nov	1200	1250	265	5
		985	122	6
		863	93	2
		770	33	2
		737	554	10
Dec	150	333	46	1
		287	57	1
		230	123	3
		107	51	1
		56	56	1
Total	3947		3947	115

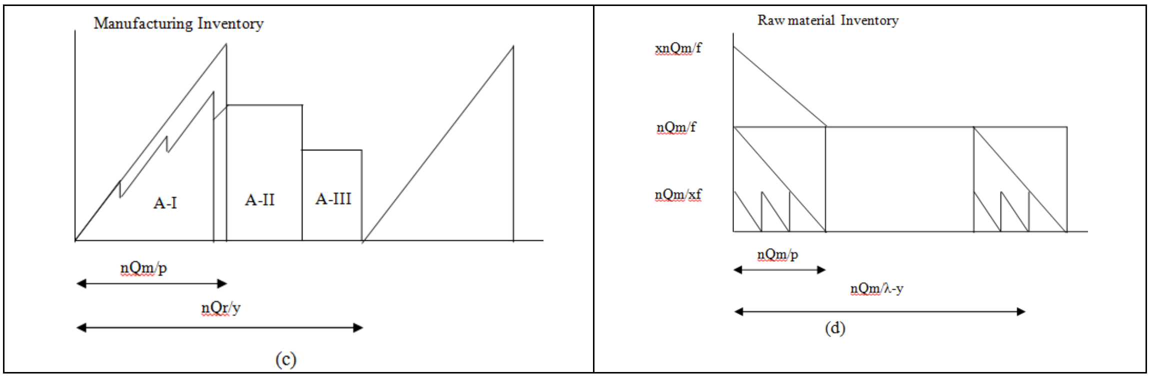

Fig. 6. Pull system inventory model

3. Model Formulation

The two models are based on the application of probability theory. Convolution and joint distribution techniques have been used to derive the distribution of lead time and marginal distribution of demand in lead time.

In the first model, the delay in placing the order has been neglected, while in second model, this delay has been considered and is taken to be distributed exponentially.

The concepts of convolution and marginal distribution have been widely discussed in (Hadley and Whitin, 1963; Cramer, 1946). The marginal distribution concept is now widely used in analyzing the inventory system, where any one or both of lead time and demand follow probability distribution pattern.

The following notation will be used to develop the model:

A — $\sqrt{\text{square of the mean of demand rate (montihly in) million) + 2 x square of standard deviation of demand rate x reciprocal of mean of administrative delay lead time.}}$ $= \sqrt{\Delta^2 + 2\sigma^2 K}$

a — Sub lot quantity in million

a_n — Net accumulation quantity in million

B_{ST} — Extra buffer stock required to eliminate stock out between inter-arrival time of sub lots

i — index of time period

B — $\sqrt{\ }$ square of the mean of demand rate + 2x square of standard deviation of demand rate x reciprocal of mean of procurement lead time = $\sqrt{\Delta^2 + 2\sigma^2 \lambda}$

c_u — Unit cost of time

c_c — Cost of carrying inventory in percentage per year

c_s — Ordering cost in Rupees per order

c(t) — Cumulated generating function of lead time demand

D — Demand in lead time

Do — Optimum Re-order point

d — Demand in any time period

d_i — Number of stock units demanded in time period i

f(D) — Distribution of demand in lead time

F(D) — Cumulative distribution of demand in lead time

g^(x) — Probability density function of administrative delay lead time

g(L) — Probability density function of procurement lead time

h — Number of standard deviation

h(T) — Probability density function of the total lead time.

H(T) — Cumulative Probability density function of h (T)

K — Reciprocal of mean of administrative delay lead time.

L — Procurement lead time

M — Minimum stock in any time period

M(t) — Moment-generating function of lead time demand

N(t) — Moment-generating function of demand

n — Number of order per year

P(s) — Probability of shortage inter-arrival time of sub lots

Q — Order quantity

Q_o — Economic order quantity

R.O.P — Re-order point

T — Total lead time

T_i — Inter-arrival time between sub lots in time period i

T_C — Total variable cost of carrying inventory

x — Administrative delay lead time

U — Loss per unit inventory if there is no demand

V — Salvage value of one unit of inventory

α — Probability of stock out in a cycle (order)

β — Permissible number of stock outs per year

λ — Reciprocal of mean of procurement lead time

Δ — Mean of demand rate of the item

Δ_a — Mean of arrival quantity of the item in T_i

Δ_n — Net accumulation of inventory in T_i

σ — Standard deviation of demand rate

σa — Standard deviation of arrival quantity

σ_n — Standard deviation of net accumulation of Inventory

$$+\frac{1}{2}\frac{C_u C_c \times 12\ \Delta M \exp(-LD)}{\beta} \tag{11}$$

Differentiating with respect to D and equating it to zero

$$\frac{L\beta C_s}{M\exp(-LD)} + C_u C_c - 6\,\frac{C_u C_c \Delta\ ML \exp(-LD)}{\beta} = 0 \tag{12}$$

$$\text{or } Q_o = \frac{1}{2}\left(\frac{2}{L} + \sqrt{\frac{4}{L^2} + \frac{8C_s}{C_u C_c} x12\Delta}\right)$$

$$= \frac{1}{L} + \sqrt{\frac{1}{L^2} + \frac{24C_s\Delta}{C_u C_c}} = \frac{1}{L}\left[1 + \sqrt{1 + \frac{24C_s\Delta L^2}{C_u C_c}}\right] \tag{13}$$

So, for any particular permissible stock out per year, one can get the Economic Order Quantity directly by the equation (13).

3.2. Model II: Demand normal, procurement lead time and administrative delay lead time exponential

This model was formulated by considering administrative delay and procurement lead times to be distributed exponentially and demand pattern distributed as a normal distribution. Resultant distribution of total lead time was first derived by applying the method of convolutions. For any order level D, to obtain the desired service level of β (permissible probability of stock outs per year) by equation (40), it was found that the order quantity Q and total cost TC are dependent on D by equation (38).

As the object is to minimize the total cost associated with holding up the order level D, a trial solution was obtained from the equation (4) by increasing D in steps and computing the corresponding Q and T.C.

3.2.1. Distribution of total lead time:

If L =Procurement lead time with mean 1/λ and exponentially distributed;

x =Administrative lead time with mean 1/K and exponentially distributed;

Then, T = Total lead time = L + x (14)

The probability density function (p.d.f.) of 1 is

$g(L) = \lambda \exp(-\lambda L) \qquad L > 0$ (15)

The p.d.f. of x is

$\hat{g}(x) = K \exp(-Kx) \qquad x > 0$ (16)

The p.d.f. of T is obtained by the law of convolution as

$$h(T) = \int_{\propto}^{+\propto} g(T - x)\,\hat{g}(x)\,dx \tag{17}$$

$$= \frac{K\lambda}{\lambda - K}\left(\exp(-KT) - \exp(-\lambda T)\right) \tag{18}$$

or $h(T) = 0 \qquad -\propto < T < 0$

$$= \frac{K\lambda}{\lambda - K}\left(\exp(-KT) - \exp(-\lambda T)\right) \quad 0 \le T < \propto \tag{19}$$

Cumulative distribution function of T is

$$H(T) = \frac{K\lambda}{\lambda - K}\int_o^T \left(\exp(-KT) - \exp(-\lambda T)\right) dT$$

$$= \frac{K\lambda}{\lambda - K}\left(\frac{1}{K}\left(1 - \exp(-KT)\right) - \frac{1}{\lambda}\left((1 - \exp(-\lambda T)\right)\right) \quad T > 0 \tag{20}$$

when T = 0, H(T) = 0

when T = ∝ H(T) = 1

3.2.2. Distribution of demand in lead time

If f(D) represents distribution of demand in lead time

$$f(D) = \int_o^{\propto} N(D, \Delta T, \sigma\sqrt{T})\ x\ h(T)\,dT$$

and $N(D, \Delta T, \sigma\sqrt{T})$ represents lead time demand distribution,

h(T) represents distribution of total lead time.

$$F(D) = \int_o^{\propto} N(D, \Delta T, \sigma\sqrt{T})\frac{K\lambda}{\lambda - K}\left(\exp(-KT) - \exp(-\lambda T)\right) dT$$

$$= \frac{K\lambda}{\lambda - K}\int_o^{\propto} N(D, \Delta T, \sigma\sqrt{T})\exp(-KT)dT$$

$$- \frac{K\lambda}{\lambda - K}\int_o^{\propto} N(D, \Delta T, \sigma\sqrt{T})\exp(-\lambda T)dT$$

$$= I_1 - I_2 \tag{21}$$

Now,

$$I_1 = \frac{K\lambda}{(\lambda - K)\sqrt{2\pi\sigma}}\int_o^{\propto} T^{-1/2}\exp\left(-\frac{(D - \Delta T)^2}{2\sigma^2 T}\right)\exp(-KT)dT$$

$$= \frac{K\lambda\exp(\Delta D/\sigma^2)}{(\lambda - K)\sqrt{2\pi}\,\sigma}\int_o^{\propto} T^{-1/2}\exp\begin{pmatrix} -\left(D^2/2\sigma^2\right)^{1/T} \\ -\left(\frac{\Delta^2}{2\sigma^2} + K\right)T \end{pmatrix} dT$$

Now, we know that

$$\int_o^{\propto} x^{\nu - 1/2}\exp\left(-\beta/x - \gamma x\right)dx =$$

$$2(\beta/\gamma)^{\nu/2}\,K_\nu\left(2\sqrt{\beta\gamma}\right) \quad \text{where } \beta > 0, \quad \gamma > 0$$

$$\text{and } K_{\pm 1/2}(Z) = \sqrt{\frac{\pi}{2z}}\,e^{-z}$$

The integrates have been given by (Gradshteyn and Ryzhik, 1965):

$$\text{So, } I_1 = \frac{K\lambda\exp(\Delta D/\sigma^2)}{(\lambda - K)\sqrt{2\pi\sigma}}\exp\left(-\frac{D}{\sigma^2}\sqrt{\left(\Delta^2 + 2\sigma^2 K\right)}\right) \tag{22}$$

when D ≥ 0

when D ≤ 0

putting D = Z when Z > 0

Again,

$$\frac{df_{-}(D)}{dD} = 0$$

or, $$\frac{\exp\left(D/\sigma^2\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)\right)}{\sqrt{\Delta^2+2\sigma^2 K}} \times \frac{\left(\sqrt{\Delta^2+2\sigma^2 K}+\Delta\right)}{2}$$

$$D = \frac{\sigma^2}{\left(\sqrt{\Delta^2+2\sigma^2 K} - \sqrt{\Delta^2+2\sigma^2 \lambda}\right)}$$

or, $$\log_e \frac{\sqrt{\Delta^2+2\sigma^2 K}\ \left(\sqrt{\Delta^2+2\sigma^2 \lambda}+\Delta\right)}{\sqrt{\Delta^2+2\sigma^2 \lambda}\ \left(\sqrt{\Delta^2+2\sigma^2 \lambda}+\Delta\right)} \quad (34)$$

So, it can be seen that the above value of D is positive; hence, no model value exists in the negative region. So, there is only one mode for the distribution of D and mode lies in the positive region.

3.2.5. Distribution function of D

For D < 0, distribution function of demand in lead time is

$$F(D) = \frac{K\lambda}{\lambda-K}\int_{\propto}^{o} \frac{\exp D/\sigma^2(A+\Delta)dD}{A} - \frac{K\lambda}{\lambda-K}\int_{-\propto}^{o} \frac{\exp(D/\sigma^2(B+\Delta))}{B}.dD$$

where A= $\sqrt{\Delta^2+2\sigma^2 K}$ and B= $\sqrt{\Delta^2+2\sigma^2 \lambda}$

or

$$F(D) = \frac{\frac{K\lambda\sigma^2}{\lambda-K}\sigma^2\exp\left(D/\sigma^2(A+\Delta)\right)}{(A+\Delta)A} - \frac{K\lambda\sigma^2\exp\left(D/\sigma^2(B+\Delta)\right)}{(\lambda-K)(\Delta+B)B}$$

$$= \frac{K\lambda\sigma^2}{\lambda-K}\frac{\exp\left(D/\sigma^2(A+\Delta)\right)}{A(A+\Delta)} - \frac{\exp\left(D/\sigma^2(B+\Delta)\right)}{B(B+\Delta)}$$

D < 0 (35)

For D > 0, the distribution function of demand in lead time is

$$F(D) = \int_{\propto}^{o} f_{-}(D)dD + \int_{o}^{D} f_{+}(D)dD$$

$$= \frac{K\lambda\sigma^2}{\lambda-K}\ \frac{1}{A(A+\Delta)} - \frac{1}{B(B+\Delta)} - \frac{K\lambda\sigma^2}{(\lambda-K)}\frac{\exp\left(-D/\sigma^2(A-\Delta)\right)}{A(A-\Delta)}$$

$$-\frac{1}{A(A-\Delta)} - \frac{\exp\left(-D/\sigma^2(B-\Delta)\right)}{B(B-\Delta)} + \frac{1}{B(B-\Delta)} \quad (36)$$

$$F(D) = \frac{K\lambda\sigma^2}{\lambda-K}\left(\begin{matrix}\frac{1}{A(A+\Delta)} - \frac{1}{B(B+\Delta)} + \\ \frac{1}{A(A-\Delta)} - \frac{1}{B(B-\Delta)}\end{matrix}\right)$$

$$= \frac{K\lambda\sigma^2}{\lambda-K} x \left(\begin{matrix}\frac{1}{\sigma^2 K} - \\ \frac{1}{\sigma^2\lambda}\end{matrix}\right) - \frac{K\lambda\sigma^2}{\lambda-K}\left(\begin{matrix}\frac{\exp\left(-D/\sigma^2(A-\Delta)\right)}{B(B-\Delta)} - \\ \frac{\exp\left(-D/\sigma^2(B-\Delta)\right)}{B(B-\Delta)}\end{matrix}\right)$$

$$= 1 - \frac{K\lambda\sigma^2}{\lambda-K}\left(\begin{matrix}\frac{\exp\left(-D/\sigma^2(A-\Delta)\right)}{A(A-\Delta)} - \\ \frac{\exp\left(-D/\sigma^2(B-\Delta)\right)}{B(B-\Delta)}\end{matrix}\right) \quad (37)$$

$$\underset{D\to\propto}{F(D)} = 1 - 0 = 1$$

3.2.6. Total cost

The total variable cost can be expressed in mathematical form:

$$\text{T.C.} = \begin{matrix}\frac{12\times\Delta}{Q}C_S + \left(D - \Delta\left(\frac{1}{K}+\frac{1}{\lambda}\right)\right) \\ C_u \times C_c + \frac{1}{2}Q \times C_u \times C_c\end{matrix} \quad (38)$$

3.2.7. Order quantity

If α be the percentage of stock out per cycle then α = 1 - F(D)

$$= \frac{K\lambda\sigma^2}{(\lambda-K)}\left(\frac{\exp(-D/\sigma^2(A-\Delta)}{A(A-\Delta)}\right) - \frac{\exp(-D/\sigma^2(B-\Delta))}{B(B-\Delta)} \quad (39)$$

If we take β = permissible total number of stock outs per year them

$$\beta = \frac{12\Delta}{Q}\times\alpha \quad (40)$$

$$\text{So } Q = \frac{12\Delta}{\beta}\times\frac{K\lambda\sigma^2}{(\lambda-K)}\left(\frac{\exp(-D/\sigma^2(A-\Delta)}{A(A-\Delta)}\right) - \frac{\exp(-D/\sigma^2(B-\Delta)}{B(B-\Delta)} \quad (41)$$

3.3. Modification of the two models when total lot is divided in sub lots

In the two models discussed before it is assumed that the total quantity is delivered at one time but in actual case

conclude that R.O.Po is dependent on procurement lead time.

7. If there is larger variation of demand rate, the R.O.Po and Qo will practically differ on nothing.
8. It is seen that as the permissible stock out/ year decreases, the R.O.Po increases rapidly.
9. When Demand rate increases, more inventories are to be purchased and R.O.Po and Qo are also increased. So, the total variable cost will also increase.
10. With increase of Administrative delay lead time, the total variable cost (T.C) is increased only slightly.
11. It is seen that with the increase of procurement lead time, the total variable cost increases.
12. There is very little variation of total variable cost with the variation of standard deviation of demand rate.
13. When permissible stock out decreases, the total variable cost increases rapidly. This is due to high level of R.O.Po and economic order quantity.

5. Conclusions

For probabilities of stock out between inter-arrival of sub lots, the modification of the results obtained by the models generates a better result. Arrival of sub lots can be dictated by the application of model. The demand rate is the constant inter-arrival time of one day.

The net accumulation (a_n) is dictated by the normal distribution with mean inter-arrival time of one day. So, there is no probability of stock out between inter-arrival of sub lots. Consequently, no correction of the results obtained by model 2 is necessary, because the critical portion is taken into account by the model.

5.1. Limitations of the Model

In an actual situation, in the future period, the demand, lead times, and the sub lots quantity may change their respective probability distribution patterns. So, in that case, the correct prediction of optimum re-order point and economic ordering quantity may be wrong by the model. In this model, the inter-arrival time between receiving of sub lots is taken constant. Actually, if it varies at an extended period randomly, the model is not correct to produce the results.

The model does not take the interaction of the other production units of the company. It does not distinguish between the different suppliers. The model does not take the effect of any rejection of sub lots due to bad qualities.

5.2. Scope of further Work

It is considered that there is scope for further work in relation to the present study with regard to the following aspects:

(i) To test the effectiveness of the inventory control system developed in this study, the actual distribution of inter-arrival time of sub lots as well as the sub lots quantity distribution can be introduced in Monte Carlo simulation model.

(ii) Taking different statistical distribution systems for demand and different lead times if conditions of the distribution change in future.

(iii) Other components may be also considered for other industries.

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