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Controlling the Bullwhip Effect in a Supply Chain Network with an Inventory Replenishment Policy Using a Robust Control Method

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Abstract

This paper develops a mathematical model using differential equations and considers a bullwhip effect in a supply chain network with multiple retailers and distributors. To ensure the stability of the entire system and reduce the bullwhip effect, a robust control method and an inventory replenishment policy are proposed. This shows that the choice of the output matrix may reduce the bullwhip effect. It was also observed that the inventory replenishment mechanism may be a negative impact on the robustness of the bullwhip effect. However, the inventory replenishment behavior may lead to the bullwhip effect on the presented model. This means that the complex supply relationships may have a significant role in controlling or reducing the bullwhip effect of fluctuations.

Keywords: Robust Control, Bullwhip Effect, Inventory Replenishment, Supply Network.

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1. Introduction

The bullwhip effect is a term used to describe progressive fluctuations in customer demand along a supply chain from downstream to upstream. Essentially, larger phenomena produce a bullwhip effect because the process of information dissemination is constantly being distorted. This has a step-by-step effect on the upstream supply chain in that manufacturers and suppliers of raw materials are supplied with distorted information and then decisions can easily lead to over-production and inventory errors. Various factors contribute to the bullwhip effect such as lead-time, type of inventory policy, and information on demand forecasting. This article focuses on retailers with an inventory replenishment policy strategy and incorporates robustness of the bullwhip effect in a supply network using parameters of uncertain demand behavior.

Chuang and Huang (2004) declared that inventory-replenishment decision is vital to the supply chain performance. The selection of an appropriate inventory policy would not only reduce the total inventory costs but also would satisfy the downstream customers and the final customer in a supply chain environment. In this aspect, most of the existing researches have been devoted the supply chain inventory decision to a single known demand distribution, such as normal or uniform. However, in a supply chain environment, there are always multiple downstream customers with different probability distributions of a demand quantity.

Daganzo (2003), Nagatani and Helbing (2004), Surana et al. (2005), Helbing et al. (2006) aimed to develop a realistic model to describe nonlinear interactions and to represent dynamics of the flow of the materials though networks. Helbing et al. (2004) proposed a supply network governed by balance equations and equations for the adaptation of production speeds and studied the stability and dynamics of supply networks. Daganzo (2004) examined the stability of multistage supply chains under arbitrary demand conditions and presented commitment-based policies that can maintain any desired inventory level for any demand rate. A supply network based on a stochastic discrete-time controlled dynamical system was proposed by Laumanns and Lefeber (2006), in which an explicit state-feedback control policy was derived to control the material flow of the supply network. Ouyang and Li (2010) analyzed the propagation and amplification of order fluctuations in supply chain networks and based on inventory management policies. They proposed robust analytical conditions to predict the presence of the bullwhip effect for any network structure. Yang et al. (2009) developed a model of a general closed-loop supply chain network and optimized the equilibrium state of the network by using the variation inequalities method. Dong et al. (2011) proposed a supply chain

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When $d_j(t)$ is constant d_j^∞, for a stable supply network, its inventory and order quantity is a constant steady state shall, $\lim_{t\to\infty} x_i(t) = x_i^\infty, \lim_{t\to\infty} x_j(t) = x_j^\infty, \lim_{t\to\infty} u_i(t) = u_i^\infty$. On this basis, the definition of the state vector is shown below:

$$x(t) = [x_1(t) - x_1^\infty, x_2(t) - x_2^\infty, \dots, x_m(t) - x_m^\infty, x_{m+1}(t) - x_{m+1}^\infty, \dots, x_{m+n}(t) - x_{m+n}^\infty]^T \quad (4)$$

In this study, in terms of the supply network, all retailers and distributors orders are quantified as a control vector. Structure control vector order and supply network are assigned a direct relationship because each distributor has its unique external suppliers, and each retailer has at least one distributor for its supply of goods, so it may be assumed that the steady-state minus the constant use of all distributors and retailers re-order quantity number to get control vector $u(t)$. Then, $u(t)$ is an order the number of vector $m + \sum_{i=1}^m \sum_{j=m+1}^{m+n} a_{ij}$. Input vector is defined by:

$$d(t) = [d_{m+1}(t) - d_{m+1}^\infty, d_{m+2}(t) - d_{m+2}^\infty, \dots, d_{m+n}(t) - d_{m+n}^\infty]^T \in R^n \quad (5)$$

In the definition of state vector, the basic control vector and the input vector on the supply network can be established by the following unified state model:

$$\begin{cases} x(t) = A_1 x(t-1) + A_2 x(t-\tau) + B_1 u(t-1) + B_2 d(t-1) \\ y(t) = Du(t) \end{cases} \quad (6)$$

where A_1, A_2 are $(m+n)\times(m+n)$ matrix, B_1 is $(m+n)\times\left(m+\sum_{i=1}^m \sum_{j=m+1}^{m+n} a_{ij}\right)$ matrix, B_2 is $(m+n)\times n$ matrix, $y(t)$ is the output vector, D is the output matrix of the system. Due to robustness of the output variable, indicators and systems are closely linked, therefore the value of the output matrix and managers expected performance of the supply network have a direct relation. For example, managers can choose a value for a distributors order set by the output matrix as the system output, focusing on suppression orders fluctuations distributor side.

3. Robustness Index and Inventory Strategy Algorithm

The main task of robust control for a variety of supply networks is to manage uncertainty. The application of a robust inventory control strategy designed node enterprises to meet the performance requirements of the system administrator. A supply network uncertainty does not consider demand as uncertainty. The uncertainty of supplying raw materials, in advance of uncertainty and structural uncertainty due to demand uncertainty is often a source of uncertainty generated by the other. This paper focuses on the robust demand environment in uncertain supply network control problems. According to Wei et al. (2013), consideration of robust indicators is determined as follows:

$$W_I = \sup_{\forall d(k)\neq 0} \left(\frac{\sum_{k=1}^\infty y(k)^T y(k)}{\sum_{k=1}^\infty d(k)^T d(k)}\right)^{1/2} \quad (7)$$

From the definition of W_I can be seen that the demand for any form, as long as the lower value of W_I, we can improve the output variables of the input variables interference ability to meet the requirements of a robust supply network if the output vector $y(t)$ contains only the distributors order quantity, and its steady-state value for the order of the matrix, W_I can be expressed by:

$$\begin{aligned} W_I &= \lim_{N\to\infty} \sup_{\forall d(k)\neq 0} \left(\frac{\sum_{i=1}^m \sum_{k=1}^N (u_i(k) - u_i^\infty)^2}{\sum_{j=m+1}^{m+n} \sum_{k=1}^N \left(d_j(k) - d_i^\infty\right)^2}\right)^{1/2} \\ &= \sup_{\forall d(k)\neq 0} \left(\frac{\sum_{k=1}^\infty Var(u_i)}{\sum_{k=1}^\infty Var(d_j)}\right)^{1/2} \end{aligned} \quad (8)$$

As can be seen, $\sum_{k=1}^\infty Var(u_i) / \sum_{k=1}^\infty Var(d_j)$ is the classic expression of the bullwhip effect, which means that demand for the unknown, as long as possible to minimize the value of W_I, distributors will be able to end the bullwhip effect, or at least to reduce it to the minimum. Specifically, in circumstances when $W_I < 1$, the bullwhip effect can completely control an entire network. It is appropriate to apply W_I bullwhip effect robustness index.

In the field of control engineering, W_I robustness index is also known as the H_∞ norm. For a control system, the goal of robust H_∞ control is through the controller design, to ensure stability of the system, based on the H_∞ norm minimization. The value of H_∞ norm, there are two methods; namely, the frequency-domain method and the time-domain method. In the frequency domain, a transfer function is based on a system. If the transfer function is $G(z)$, then its H_∞ norm is $max\left|G(e^{j\omega})\right|_{\omega\in(0,2\pi)}$, calculation of the time domain by defining a suitable Lyapunov function, and a robust control problem into the LMI problem solving. This idea in a robust control theory is widely used.

Since this construct is a state-space model for calculating time-domain method, robustness indicators used to optimize robustness of the indicator optimization problem can be transformed into a linear matrix inequality constraints due to Wei et al. (2013). This gives a general model of robust control algorithms, in which direct reference to ideas in the literature present the following conclusions.

$$B_1 = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{13}$$

Considering the existence of inventory replenishment between behavior and output matrix according to the retailer, different values of D are controlled according to four cases.

Case 1: Select the retailer's order quantity from output vectors (i.e., $D = diag\ (I_{A1}, I_{B1})$) among I_{A1} value of all the elements 0 three square matrix and I_{B1} for the fifth-order unit matrix. Furthermore, it is assumed $A_2 = 0$, that behavior is not considered as inventory replenishment among retailers.

Case 2: Select output variables and shapes the same situation, $D = diag\ (I_{A2}, I_{B2})$, in which I_{A2} there are elements of the value taken 0 to 3 order of party array, I_{B2} as 5-order unit matrix; consider inventory replenishment strategies among retailers, inventory replenishment between the retailer for delay $\tau = 2$, set $r_{45} = r_{54} = 0.0025$, $r_{56} = r_{65} = 0.0015$, $r_{56} = r_{65} = 0.005$, $r_{78} = r_{87} = 0.002$.

Case 3: Select distributors order quantity from the output vector $D = diag\ (I_{A3}, I_{B3})$, among I_{A3} as 3 order unit matrix, I_{B3} for all elements 0 to 5 square matrix. There is no consideration for inventory replenishment between retailers, namely $A_2 = 0$.

Case 4: Select output variables and circumstances 3 the same, that $D = diag\ (I_{A4}, I_{B4})$, among I_{A4} as 3 order unit matrix, I_{B4} for all elements 0 of 5 square matrix and circumstances 3 different strategies are considered for inventory replenishment among retailers, inventory replenishment between the retailer for delay $\tau = 2$, set $r_{45} = r_{54} = 0.0025$, $r_{56} = r_{65} = 0.0015$, $r_{56} = r_{65} = 0.005$, $r_{78} = r_{87} = 0.002$.

According to Eqs. (7) and (8), an optimization algorithm using by the MATLAB of LMI toolbox is considered to determine value of the inventory control policy, in which γ for four different above-mentioned cases is minimized. Corresponding to these cases, the minimum value of γ is shown in Table 1.

Table 1
Minimum value of γ per each case

	Case 1	Case 2	Case 3	Case 4
Min of γ	4.0	6.355	5.656	7.514

This table also shows an inventory replenishment policy strategy does not significantly improve robustness of the bullwhip effect in supply networks. In order to further reveal the advantages of robust control methods, using the specific needs of the bullwhip effect model to study the particular problem before simulation needs to apply the bullwhip effect, retailers and distributors are given the bullwhip effect metric expression.

$$BW_R = \frac{\sum_{j\in N_R} Var(u_j(t))}{\sum_{j\in N_R} Var(d_j(t))} \tag{14}$$

$$BW_D = \frac{\sum_{i\in N_D} Var(u_j(t))}{\sum_{j\in N_R} Var(d_j(t))} \tag{15}$$

BW_R and BW_D are represented retailers and distributors bullwhip effect expression is worth noting that in numerical simulation, variance calculation and simulation length.

5. Results and Discussion

When demand obedience considers 20 retailers, variance 100 under normal distribution, Fig. 2 shows that in Case 4 there is a steady state under three distributors on the dynamic curve. This figure also shows that based on robust H_∞ control of the supply network design strategies to ensure stability of the order of the entire supply network helped reduce ordering volatility curve which can order to a steady state at a faster rate. It means that the system can respond more quickly to customer needs. In addition, Fig. 3 shows the convergence curve for the

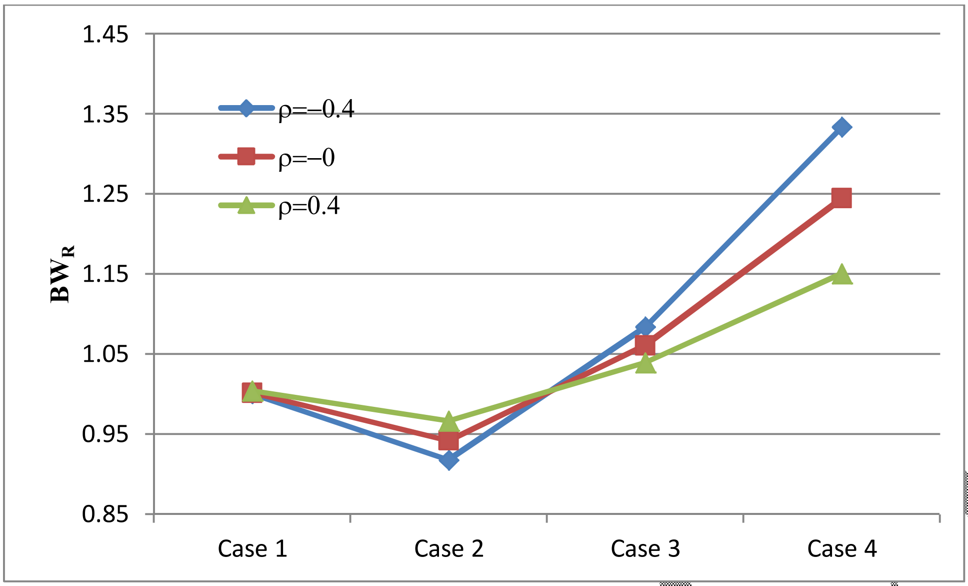

Fig. 4. Bullwhip effect BW_R per each case and three correlation coefficients

From Figs. 3 and 4 and Table 2, the following conclusions can be drawn: 1) Comparison of cases shows that in Cases 1 and 3 in terms of the bullwhip effect, the value of retailers and distributors bullwhip effects have a significant number. Cases 2 and 4 are compared to the situation bullwhip effect from end retailers to smaller. Its distributors bullwhip effect shows a significant number, which means the robust control design strategies for an inventory can be targeted to select some quantities from companies orders as an output variable to reduce the effect of these enterprises on the bullwhip effect. 2) Cases 1 and 2 compared with Cases 3 and 4, there is an amount of the order retailer output variables and inventory replenishment strategies help reduce the bullwhip effect in the supply networks. When selecting distributors, order quantity of output variables, the inventory replenishment strategy helps to alleviate the bullwhip effect in distribution but fluctuations are caused by retailers in the orders found in front of the inventory replenishment strategy, although the bullwhip effect does not necessarily enhance robustness of the supply network, it was able to suppress the bullwhip effect nicely under specific demand model. 3) Whether customer needs or demands are positively correlated or negatively related to the design of robust control method for inventory strategies helps to curb the overall bullwhip effect in a supply network and improve the speed of a response of the supply network.

6. Conclusion

A two-echelon supply network was modeled for retailers and distributors with an application of robust H_∞ control method designed according to ordering policies at each node. Studies showed that the application of robust control for inventories functions to control the bullwhip effect might well inhibit a supply network by setting the output matrix. It could be targeted to inhibit the target node bullwhip effect and reduced production costs and inventory costs. The results of numerical simulation showed that the strategy of inventory replenishment, although not significantly, might reduce supply bullwhip effect of a network, but demand for a particular form, can be effective in terms of inhibiting the bullwhip effect. This study described the complex supply relationships that might be an important factor in inhibiting the bullwhip effect in a supply network research; however, some shortcomings remained. Moreover, considering only two levels of a supply network, the structure in reality a supply network had more complexity.

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