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Developing Fuzzy Tool Capability Measurement System Analysis

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Abstract

Due to the existing competitive environment of global economy, many companies allocate their major financial and human resources to quality improvement. Since, using measurement tool is an essential component in quality analysis and highly depends on quality of measurement systems and their results; measurement systems applications after calibration are the most efficient methods in real operations. Thus, inherent changes of measurement tools can be studied by computing capability index of measurement tools. This paper aims to develop a fuzzy model for computing capability. Fuzzy model for computing capability (C_g ،C_{gk}) with data in the form of fuzzy triangular and trapezoidal numbers using MATLAB is developed and then a case study applying proposed method is presented. Finally, we compare presented results with classical outcomes and prove that fuzzy environment gives more flexibility, rather than classical environment.

Keywords: Measurement system analysis; Tool capability indexes; Fuzzy numbers.

1. Introduction

In recent years, there have been considerable pressure result's from competitive environment thus from a global economic point of view, it affects the quality aspects of industrial companies. Statistical methods have been introduced as helpful methods for quality control. Furthermore, qualitative changes in a product are known as crucial problems, since human being and machine are main drivers of manufacturing processes. Thus, statistical quality control methods are needed until we face these problems (Faraz & Shapiro, 2010).

Nowadays, values derived from measurement precedures are utilised in the form of various terms and are strongly prevalent rather than before. Quality of products can be assessed by means of statistics deduced from manufacturing processes. Thus, decision making highly depends on quality of measurement precedure. In other words, exploiting measurement analysis totally deals with quality of measurement system. If quality of measurement system and quality of results are low, process procedure would not be valid. Calibrasion has been far a reliable method for estimating sensitivity of measurement devices. In calibration, measurement devices are assessed in ideal condition such as isolated measurement units, standard prosedures and under supervision by experts. In sense, it is not a helpful method for acquiring capability of systems in real world. Therefore, efficiency of measurement systems can be achieved with measurement systems analysis in following situations:

- When a measurement device or gage is used in real location.
- When a measurement device has multiple users.
- When a real part is under measurement procedure.
- When application environment is not robust.
- When a measurement device is used continuously.

Therefor, calibration is not solely enough to control precision and accuracy of measurement devices in real world. There is a chapter in DIN EN ISO9000 entitled 'Inspection, Measuring and Test Equipment' that highlights capabilities of measurement systems. In 1989, General Motors was known as a pioneer company that developed a guidebook in this area. Later, Ford Motor Company also presented other guides. Other guidebook was published by Robert Bosch Group under the title of 'achieving capability of measurement system in reality' in 1990, as well as Mercedes Benz that introduced another guidebook.

Components of measurement system analysis specify whether a measurement system is acceptable, marginal or unacceptable, and it can be applied for:

- Assessment of devices or new measurement methods
- Comparison of measurement device performance before and after adjustment
- Comparison of total inspection measurement methods of side contractors and measurement method of row materials inspection
- Creating a fundamental approach to compare two measurement devices
- Establishing a proper method to ensure accuracy and reliability of manufacturing process
- Determining capability of measurement system
- Creating a basis to obtain calibration time of

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measurement device

Calculating capability index of a measurement device enables measuring any existing variations in measurement devices. Using C_g and C_{gk}, repetition and propensity could be evaluated simultaneously. They are applied to new and repaired devices as well as measurement methods confirmation. The formula for calculating the C_g and C_{gk} is shown in Table (1) (Kazemi et al., 2010).

Table 1
The formula for calculating the C_g and C_{gk}

Index	According to tolerance	According to process
C_g	$C_g = \frac{0.2T}{6S_g}$	$C_g = \frac{0.15(6\sigma_p)}{6S_g} = \frac{0.15\sigma_p}{S_g}$
C_{gk}	$C_{gk} = \frac{0.1T - \lvert \bar{X}_g - X_m \rvert}{3S_g}$	$C_{gk} = \frac{o.5 \times 0.15(6\sigma_p) - \lvert \bar{X}_g - X_m \rvert}{3S_g} = \frac{0.45\sigma_p - \lvert \bar{X}_g - X_m \rvert}{3S_g}$
Minimum acceptance criteria	1.33	1

In Table (1), T is range of tolerance, S_g is deviation from observed values, σ_g is variance and m is sample size. In some cases, confronting with vague and uncertain information leads to impair efficiency of these indexes. Although, there have been proposed solutions in this regard but fuzzy logic is recognized as a leading method in this area. Black and white thinking has been always criticized. Even though definite science was able to explain most of phenomenon correctly, classic logic has presented appropriate conclusions. On the other hand, they are not capable to model, describe and express our environment (Ghazanfari & Rezaee, 2006). Thus, fuzzy logic that is introduced by Zadeh (Zadeh, 1965), is applicable in uncertain environment of industries (Bojadziev & Bojadziev, 1991). Fuzzy numbers are used instead of crisp numbers because there is usually uncertainty in the measurement system.

1.1. Fuzzy membership functions

In the literature, theory of fuzzy sets, some fuzzy membership one presented. The most widely used is triangular and trapezoidal membership function. We used triangular and trapezoidal fuzzy numbers in this paper because they are more applicable to industries than other fuzzy membership functions.

1.1.1. Triangular membership function

Triangular membership function (TRIMF) defined by three parameters a, b and c and the equation (1) is calculated.

$$f(x,a,b,c) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c \\ 0 & c \le x \end{cases} \qquad (1)$$

Triangular membership function in Figure (1) is shown.

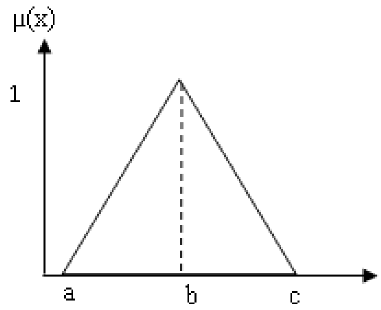

Fig. 1. Triangular membership function

1.1.2. Trapezoidal membership function

Trapezoidal membership function (TRAPMF), defined by four parameters a, b, c and d and the equation (2) is calculated.

$$f(x,a,b,c,d) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{d-c} & c \le x \le d \\ 0 & d \le x \end{cases} \qquad (2)$$

Trapezoidal membership function in Figure (2) is shown.

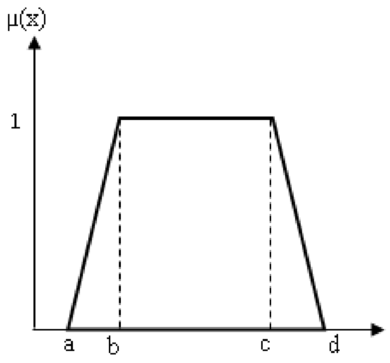

Fig. 2. Trapezoidal membership function

1.2. Background of research in measurement systems analysis

In 1993, Montgomery and Runger discussed that measurement systems play an important role in quality improvement of organizations (Montgomery & Runger, 1993). Concurrently, they displayed weak and strong points of modified analysis of variance using a nested design and factorial design in addition to gauge reproducibility and repeatability index (GR&R) of device and confidence interval (Mandel, 1997).

Burdick et al. state that while inspectors are fixed, calculations associated with reproducibility and repeatability index (GR&R) are more appropriate while using hybrid method (Burdick et al., 2002).

In 2004, three ways for obtaining reproducibility and repeatability index (GR&R) were proposed: analysis of variance model, classic model and long form (Pan, 2004). Lee classifies measurement errors in three groups: measurement equipment, inspectors and environment (Lee, 2005). Ostadsharifmemar and Akhavan Niaki the available single charting methods, which have been proposed to detect simultaneous shifts in a single process mean and variance, are investigated. Then, by designing proper simulation studies these methods are evaluated in terms of in-control and out-of control average run length criteria (ARL) (Ostadsharifmemar and Akhavan Niaki, 2008). In 2005, it is claimed that counting reproducibility and repeatability index (GR&R) enables inspectors to analyze measurement system. But it is not advisable when there are interactions between inspectors and departments, since accurate information cannot be provided (Fang &Wang, 2005). Fang et al. followed study of Fang and Yang (2005) and detected a significant deviation in estimating variance using classic and long form model (Fang et al., 2009). Al-Refaie and Bata pointed out that applying reproducibility and repeatability index (GR&R) to evaluate measurability in manufacturing process, is perceived as a form of analysis of variance (Refaie, 2010). Repeatability analysis indicates various capacities of measurement, whilst reproducibility analysis identifies whether various factors of measurement are available or not.

Runje et al., described the procedures for the measurement system analysis in the manufacturing process and mathematical background of implemented steps. Analysis of the measurement system is different from case to case, and depends on the number of operators, the number of measured parts and number of replicates (Runje et al., 2017). Mahshid et al., in their paper, presented a new statistical analysis for manufactured products in order to evaluate the degree of tolerance obtained when the process is identified while using the capability and uncertainty ratio of the expansion. This analysis has advantages for process planning, precision constraints, and process optimization. Since this is dependent on the measurement, eliminating any potential errors has a significant negative impact on the results. Therefore, measurement uncertainty is used in combination with the process capability ratio to determine compliance and non-compliance with the requirements related to the qualitative characteristics of the work component population (Mahshid et al., 2018). Cepova et al., focused on a detailed explanation of the average and range method (Automotive Industry Action Group, Measurement System Analysis approach) and of the honest Gauge Repeatability and Reproducibility method (Evaluating the Measurement Process approach). The measured data (thickness of plastic parts) were evaluated by both methods and their results were compared on the basis of numerical evaluation. Both methods were additionally compared and their advantages and disadvantages were discussed (Cepova et al., 2018).In general, many of the tools described in the scientific literature, set by international standards and / or in practical applications, are based on the normal assumption of information distribution. This involves some alternative approaches to "R&R reading" when distributing anomalous data. To solve this problem in practical applications, Genta and Galetto have used computer simulation (Genta and Galetto, 2018).Process capability analysis, which can be defined as the capability of any process to meet customer demands expressed through specification constraints. Processes can be classified as "power" and "powerless" according to the values of process capability indices. Therefore, PCA is widely used and has important effects on the production process. Fuzzy set theory can be successfully applied to counter ambiguity and add more flexibility and sensitivity to traditional PCIs. To this end, upper and lower profile constraints can be expressed using linguistic variables. Fuzzy Process Capability Indicators (FPCIs) can be generated using the mean and fuzzy variance. There are many studies using FPCI for PCA in the literature. Kaya and Colak analysed these studies according to some of their characteristics such as year, type of document, journal name, and country. Also classifications including FPCI, application area, fuzzy parameters and type of fuzzy sets are presented in this study. The main purpose of this article is to illustrate possible future research areas for fuzzy PCA (Kaya and Colak, 2020).

1.3. Background of research on fuzzy measurement systems analysis

There have been studies about calculating reproducibility and repeatability in fuzzy environment for measurement system analysis. Kazemi et al. dedicated a table on accuracy and correctness of measurement systems using triangular fuzzy numbers to evaluate reproducibility and repeatability (GR&R) (Kazemi et al., 2010).They also developed an expert system approach to the study and implemented trapezoidal fuzzy numbers (Hajipour et al., 2013). Yeh et al. investigated number of distinct categories (NDC) using fuzzy logic (Yeh et al., 2015).

Based on the reviewed literature, it is observed that all the accomplished works focuses on GR&R and NDC in MSA but there is still a lack of measuring tool capability in an uncertain environment. Moreover, MSA is recommended by ISO/TS16949 as an international standard for automotive industry, the importance of tool capability is highlighted. Hence, the research question carried out here: How is the capability of tools in MSA?

In this context, the objective of the research established as follows:

1. To develop FMSA model for tools capability
2. To measure the efficiency of tool capability in fuzzy and classical environment

Therefore, research framework is developed as shown in Figure 3. The framework is divided to four levels. The first level is developing the MSA fuzzy model. Later, experts need to collect required data and measure the MSA indices in two situations as per described by crisp and fuzzy. The forth level is comparison and conclusion between two models.

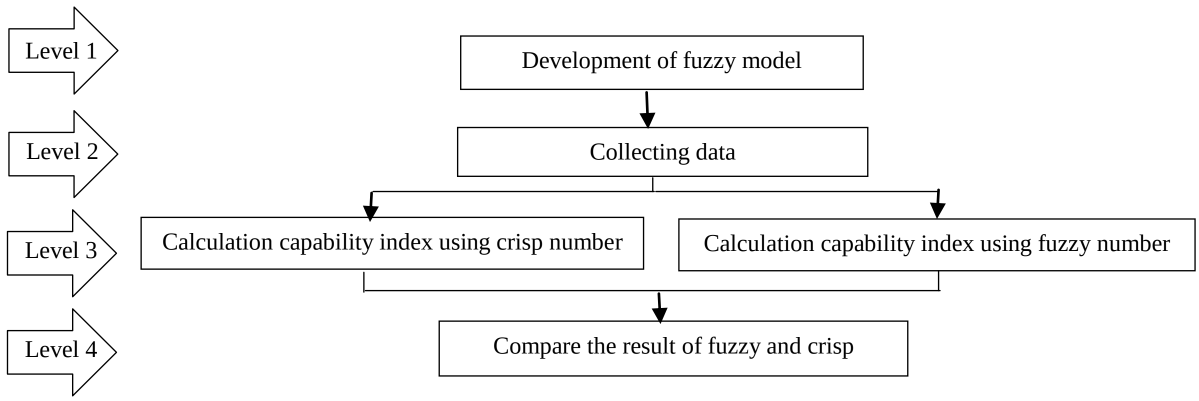

Fig. 3. Research framework

2. Research Method

In calculating measurement system capability, $\bar{X}_g$ and S_g are mean and standard deviation of observed data gathered by measurement device. They are counted using equations (3) and (4).

$$\bar{X}_g = \frac{1}{n}\sum_{i=1}^{n} X_i \quad (3)$$

$$S_g = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X}g)^2}{n-1}} \quad (4)$$

2.1. Model development using triangular fuzzy numbers

Where measurement system produces X, it can be shown as a triangular fuzzy number in relation (5). Having been measured a sample for n times; mean can be calculated as relation (6):

$$X_i = (X_1, X_2, X_3) \quad (5)$$

$$\bar{X}_g = \left(\frac{\sum_{i=1}^{n} X_{i1}}{n}, \frac{\sum_{i=1}^{n} X_{i2}}{n}, \frac{\sum_{i=1}^{n} X_{i3}}{n}\right) = TFN(\bar{X}_{g1}, \bar{X}_{g2}, \bar{X}_{g3}) \quad (6)$$

Standard deviation can be computed in fuzzy environment as following (7).

$$\tilde{S}_g = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X}_g)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}[(X_{i1}, X_{i2}, X_{i3}) - (\overline{X}_{g1}, \overline{X}_{g2}, \overline{X}_{g3})]^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}[(X_{i1} - \overline{X}_{g3}, X_{i2} - \overline{X}_{g2}, X_{i3} - \overline{X}_{g1})]^2}{n-1}} = TFN(\tilde{S}_{g1}, \tilde{S}_{g2}, \tilde{S}_{g3}) \quad (7)$$

2.1.1. Device capability index C_g based on tolerance range using triangular fuzzy numbers

In this phase, capability index C_g is developed using triangular fuzzy numbers. According to Table (1) C_g is calculated with fuzzy number (equation (8)).

$$C_g = \frac{0/2(T)}{6\tilde{S}_g} = \frac{0/2(T_1, T_2, T_3)}{6(S_{g1}, S_{g2}, S_{g3})} = \frac{(0/2T_1, 0/2T_2, 0/2T_3)}{(6S_{g1}, 6S_{g2}, 6S_{g3})} = TFN(C_{g1}, C_{g2}, C_{g3}) \quad (8)$$

2.1.2. Device capability index C_{gk} based on tolerance range using triangular fuzzy numbers

Where C_{gk} is displayed as a fuzzy number using equation (9).

$$C_{gk} = \frac{0/1T - \left|\overline{X}_g - X_m\right|}{3\tilde{S}_g} = \frac{0/1(T_1, T_2, T_3) - \left|(\overline{X}_{g1}, \overline{X}_{g2}, \overline{X}_{g3}) - (X_{m1}, X_{m2}, X_{m3})\right|}{3(S_{g1}, S_{g2}, S_{g3})} = \frac{(0/1T_1, 0/1T_2, 0/1T_3) - \left|(\overline{X}_{g1} - X_{m3}, \overline{X}_{g2} - X_{m2}, \overline{X}_{g3} - X_{m1})\right|}{(3S_{g1}, 3S_{g2}, 3S_{g3})} = TFN(C_{g1}, C_{g2}, C_{g3}) \quad (9)$$

2.1.3. Device capability index C_g based on process range using triangular fuzzy numbers

If there is an active variance$(6\sigma_p)$, it can be applied in relations to calculate device capability index. which is presented as a triangular fuzzy number in relation (10).

$$C_g = \frac{0/15(\sigma_p)}{\tilde{S}_g} = \frac{0/15(\sigma_{p1}, \sigma_{p2}, \sigma_{p3})}{(S_{g1}, S_{g2}, S_{g3})} = \frac{(0/15\sigma_{p1}, 0/15\sigma_{p2}, 0/15\sigma_{p3})}{(S_{g1}, S_{g2}, S_{g3})} = (\frac{0/15\sigma_{p1}}{S_{g1}}, \frac{0/15\sigma_{p2}}{S_{g2}}, \frac{0/15\sigma_{p3}}{S_{g3}}) = TFN(C_{g1}, C_{g2}, C_{g3}) \quad (10)$$

2.1.4. Device capability index C_{gk} based on process range using triangular fuzzy numbers

Device capability index C_{gk} is presented in Table (1) and it is based on process range using triangular fuzzy numbers in relation (11). Minimun acceptance range for C_g and C_{gk} is 1.33 based on tolerance range and is based on process range. Values gained from above relations suggest device capability.

$$C_{gk} = \frac{0/45\sigma_p - \left|\overline{X}_g - X_m\right|}{3\tilde{S}_g} = \frac{0/45(\sigma_{p1}, \sigma_{p2}, \sigma_{p3}) - \left|(\overline{X}_{g1}, \overline{X}_{g2}, \overline{X}_{g3}) - (X_{m1}, X_{m2}, X_{m3})\right|}{3(S_{g1}, S_{g2}, S_{g3})} = \frac{(0/45\sigma_{p1}, 0/45\sigma_{p2}, 0/45\sigma_{p3}) - \left|(\overline{X}_{g1} - X_{m3}, \overline{X}_{g2} - X_{m2}, \overline{X}_{g3} - X_{m1})\right|}{(3S_{g1}, 3S_{g2}, 3S_{g3})} = TFN(C_{g1}, C_{g2}, C_{g3}) \quad (11)$$

2.2. Model development using fuzzy trapezoidal numbers

Trapezoidal fuzzy numbers in the measurement system can be represented by the relation (12). The average evaluation of the samples for n times is calculated using (13). The standard deviation is calculated by trapezoidal fuzzy numbers using the relation (14).

$$X_i = (X_1, X_2, X_3, X_4) \tag{12}$$

$$\overline{X}_g = \left(\frac{\sum_{i=1}^{n} X_{i1}}{n}, \frac{\sum_{i=1}^{n} X_{i2}}{n}, \frac{\sum_{i=1}^{n} X_{i3}}{n}, \frac{\sum_{i=1}^{n} X_{i4}}{n}\right) = \mathrm{TrFN}(\overline{X}_{g1}, \overline{X}_{g2}, \overline{X}_{g3}, \overline{X}_{g4}) \tag{13}$$

$$\tilde{S}_g = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X}_g)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}[(X_{i1}, X_{i2}, X_{i3}, X_{i4}) - (\overline{X}_{g1}, \overline{X}_{g2}, \overline{X}_{g3}, \overline{X}_{g4})]^2}{n-1}}$$
$$= \sqrt{\frac{\sum_{i=1}^{n}[(X_{i1} - \overline{X}_{g4}, X_{i2} - \overline{X}_{g3}, X_{i3} - \overline{X}_{g2}, X_{i4} - \overline{X}_{g1})]^2}{n-1}} = \mathrm{TFN}(\tilde{S}_{g1}, \tilde{S}_{g2}, \tilde{S}_{g3}, \tilde{S}_{g4}) \tag{14}$$

2.2.1. Device capability index C_g based on tolerance range using trapezoidal fuzzy numbers

In this phase, capability index C_g is developed based on tolerance range using trapezoidal fuzzy numbers that is shown in relation (15).

$$C_g = \frac{0/2(T)}{6\tilde{S}_g} = \frac{0/2(T_1, T_2, T_3, T_4)}{6(S_{g1}, S_{g2}, S_{g3}, S_{g4})} = \frac{(0/2T_1, 0/2T_2, 0/2T_3, 0/2T_4)}{(6S_{g1}, 6S_{g2}, 6S_{g3}, 6S_{g4})} = \mathrm{TrFN}(C_{g1}, C_{g2}, C_{g3}, C_{g4}) \tag{15}$$

2.2.2. Device capability index C_{gk} based on tolerance range using trapezoidal fuzzy numbers

Device capability index C_{gk} is developed in equation (16) using trapezoidal fuzzy numbers.

$$C_{gk} = \frac{0/1T - \left|\overline{X}_g - X_m\right|}{3\tilde{S}_g} = \frac{0/1(T_1, T_2, T_3, T_4) - \left|(\overline{X}_{g1}, \overline{X}_{g2}, \overline{X}_{g3}, \overline{X}_{g4}) - (X_{m1}, X_{m2}, X_{m3}, X_{m4})\right|}{6(S_{g1}, S_{g2}, S_{g3}, S_{g4})}$$
$$= \frac{(0/1T_1, 0/1T_2, 0/1T_3, 0/1T_4) - \left|(\overline{X}_{g1} - X_{m4}, \overline{X}_{g2} - X_{m3}, \overline{X}_{g3} - X_{m2}, \overline{X}_{g4} - X_{m1})\right|}{(6S_{g1}, 6S_{g2}, 6S_{g3}, 6S_{g4})}$$
$$= \mathrm{TrFN}(C_{g1}, C_{g2}, C_{g3}, C_{g4}) \tag{16}$$

2.2.3. Device capability index C_g based on process range using trapezoidal fuzzy numbers

Device capability index C_g is calculated in equation (17) using trapezoidal fuzzy numbers.

$$C_g = \frac{0/15(\sigma_p)}{\tilde{S}_g} = \frac{0/15(\sigma_{p1}, \sigma_{p2}, \sigma_{p3}, \sigma_{p4})}{(S_{g1}, S_{g2}, S_{g3}, S_{g4})} = \frac{(0/15\sigma_{p1}, 0/15\sigma_{p2}, 0/15\sigma_{p3}, 0/15\sigma_{p4})}{(S_{g1}, S_{g2}, S_{g3}, S_{g4})}$$
$$= \left(\frac{0/15\sigma_{p1}}{S_{g1}}, \frac{0/15\sigma_{p2}}{S_{g2}}, \frac{0/15\sigma_{p3}}{S_{g3}}, \frac{0/15\sigma_{p4}}{S_{g4}}\right) = \mathrm{TrFN}(C_{g1}, C_{g2}, C_{g3}, C_{g4}) \tag{17}$$

2.2.4. Device capability index C_{gk} based on process range using trapezoidal fuzzy numbers

Device capability index C_{gk} is computed in equation (18) using trapezoidal fuzzy numbers. Values concluded from above equations suggest device capability.

$$\begin{aligned} C_{gk} &= \frac{0/45\sigma_p - \left|\overline{X}_g - X_m\right|}{3\tilde{S}_g} \\ &= \frac{0/45(\sigma_{p1},\sigma_{p2},\sigma_{p3},\sigma_{p4}) - \left|(\overline{X}_{g1},\overline{X}_{g2},\overline{X}_{g3},\overline{X}_{g4}) - (X_{m1},X_{m2},X_{m3},X_{m4})\right|}{3(S_{g1},S_{g2},S_{g3},S_{g4})} \\ &= \frac{(0/45\sigma_{p1},0/45\sigma_{p2},0/45\sigma_{p3},0/45\sigma_{p4}) - \left|(\overline{X}_{g1}-X_{m4},\overline{X}_{g2}-X_{m3},\overline{X}_{g3}-X_{m2},\overline{X}_{g4}-X_{m1})\right|}{(3S_{g1},3S_{g2},3S_{g3},3S_{g4})} \\ &= TrFN(C_{g1},C_{g2},C_{g3},C_{g4}) \end{aligned} \qquad (18)$$

3. Results and Discussion

In this study, supply chain of SAIP Company in quality control department of Kiyan Khodro is considered as a population and data Supply chain of automotive industry Iran are collected. A micrometer with three spindles (Figure 4) for assessing clutch cover parts (Figure 5), considering initial lever's dimension with map tolerance of (50-50, 0.25), in 25 groups with 4 observations.

Actual environment of the experiment is the quality control department of the company, where all measurement procedures are accomplished by professional personnel.
To accomplish measurement procedure, a part is chosen as a reference part that is not replaced during the experiment. Then, measurement is done 25 times.

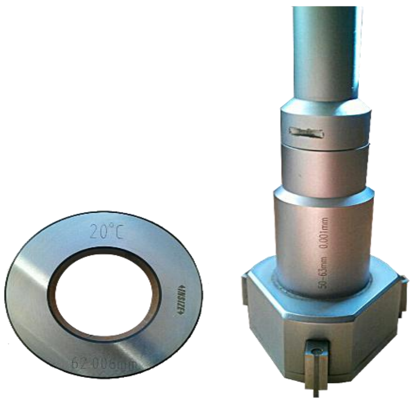

Fig. 4. Micrometer

Fig. 5. Clutch cover part

Run chart for random data that is shown in Figure (6).

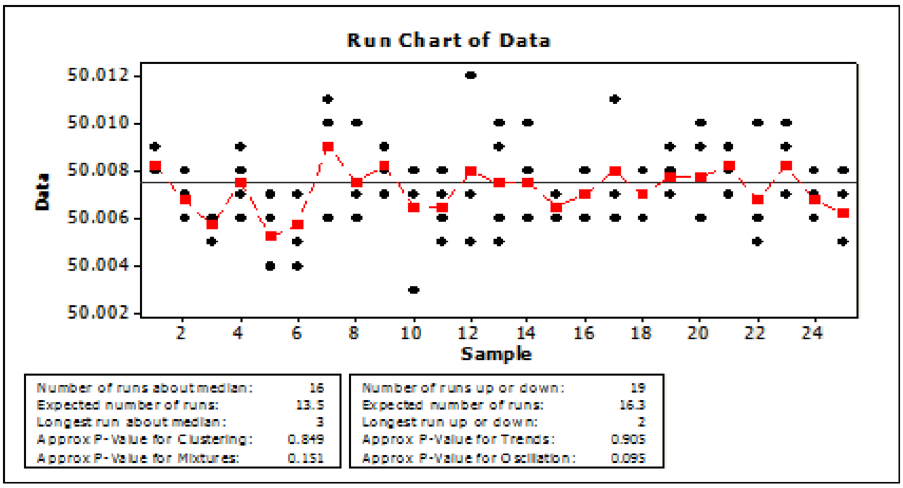

Fig. 6. Run chart

3.1. Industrial data in form of triangular fuzzy numbers

Table (2) presents triangular fuzzy data.

Table 2
triangular fuzzy numbers

NO.	x_1	x_2	x_3	x_4
1	50/007,50/008,50/009	50/005,50/006,50/007	50/008,50/009,50/010	50/007,50/008,50/009
2	50/008,50/009,50/010	50/009,50/010,50/011	50/004,50/005,50/006	50/005,50/006,50/007
3	50/007,50/008,50/009	50/010,50/011,50/012	50/005,50/006,50/007	50/008,50/009,50/010
4	50/007,50/008,50/009	50/006,50/007,50/008	50/009,50/010,50/011	50/009,50/010,50/011
5	50/005,50/006,50/007	50/005,50/006,50/007	50/007,50/008,50/009	50/005,50/006,50/007
6	50/007,50/008,50/009	50/009,50/010,50/011	50/005,50/006,50/007	50/006,50/007,50/008
7	50/006,50/007,50/008	50/006,50/007,50/008	50/006,50/007,50/008	50/008,50/009,50/010
8	50/005,50/006,50/007	50/006,50/007,50/008	50/005,50/006,50/007	50/007,50/008,50/009
9	50/005,50/006,50/007	50/008,50/009,50/010	50/005,50/006,50/007	50/008,50/009,50/010
10	50/005,50/006,50/007	50/008,50/009,50/010	50/006,50/007,50/008	50/004,50/005,50/006
11	50/004,50/005,50/006	50/007,50/008,50/009	50/007,50/008,50/009	50/009,50/010,50/011
12	50/005,50/006,50/007	50/002,50/003,50/004	50/006,50/007,50/008	50/005,50/006,50/007
13	50/006,50/007,50/008	50/006,50/007,50/008	50/005,50/006,50/007	50/005,50/006,50/007
14	50/008,50/009,50/010	50/007,50/008,50/009	50/006,50/007,50/008	50/008,50/009,50/010
15	50/007,50/008,50/009	50/007,50/008,50/009	50/005,50/006,50/007	50/009,50/010,50/011
16	50/005,50/006,50/007	50/004,50/005,50/006	50/006,50/007,50/008	50/006,50/007,50/008
17	50/005,50/006,50/007	50/006,50/007,50/008	50/010,50/011,50/012	50/006,50/007,50/008
18	50/006,50/007,50/008	50/005,50/006,50/007	50/007,50/008,50/009	50/006,50/007,50/008
18	50/003,50/004,50/005	50/007,50/008,50/009	50/006,50/007,50/008	50/007,50/008,50/009
20	50/003,50/004,50/005	50/006,50/007,50/008	50/005,50/006,50/007	50/005,50/006,50/007
21	50/003,50/004,50/005	50/004,50/005,50/006	50/006,50/007,50/008	50/005,50/006,50/007
22	50/004,50/005,50/006	50/007,50/008,50/009	50/007,50/008,50/009	50/004,50/005,50/006
23	50/006,50/007,50/008	50/011,50/012,50/013	50/006,50/007,50/008	50/006,50/007,50/008
24	50/006,50/007,50/008	50/009,50/010,50/011	50/006,50/007,50/008	50/004,50/005,50/006
25	50/008,50/009,50/010	50/005,50/006,50/007	50/008,50/009,50/010	50/007,50/008,50/009

Table 3 presents required parameters for problem solution.

Table 3
Parameters for problem solution

parameters	
(50/0063,50/0073,50/0083)	$\bar{X}=(\bar{X}_1,\bar{X}_2,\bar{X}_3)$
(0/0016,0/0017,0/0018)	VAR=(VAR, VAR, VAR_3)
(0/011,0/012,0/013)	T=(T_1, T_2, T_3)

3.1.1. Device capability indexes calulations based on tolerance range using triangular fuzzy numbers

Table (4) indicates values of device capability indexes based on tolerance range using triangular fuzzy numbers. Results of capability indexes based on tolerance range using triangular fuzzy numbers are shown in Figure (7).

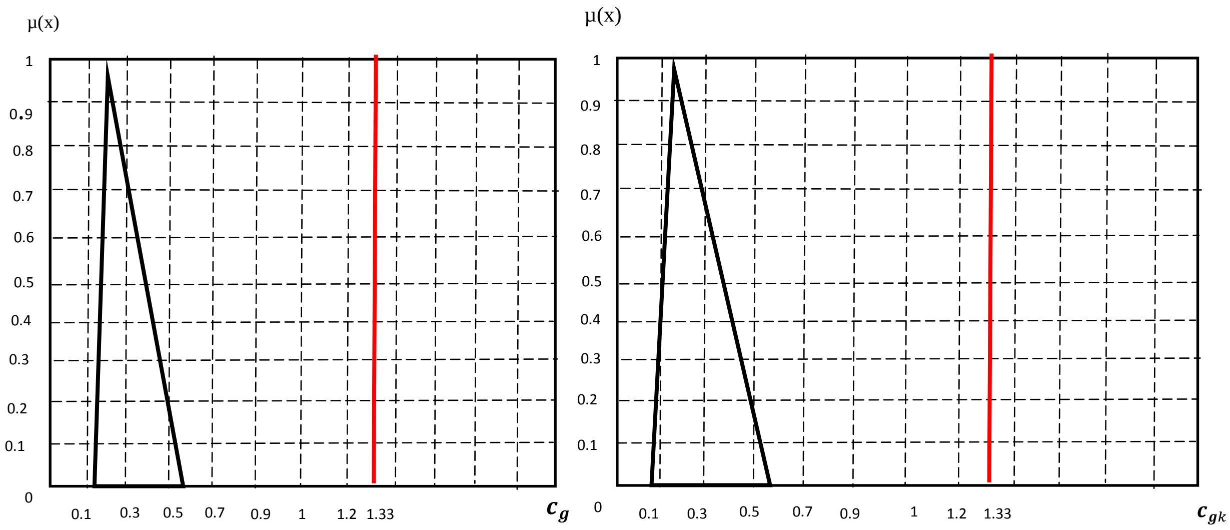

Fig. 7. capability indexes based on tolerance range using triangular fuzzy numbers

Table 4
Device capability indexes based on tolerance range using triangular fuzzy numbers

capability index	
(0/1253,0/2398,0/5746)	c_g
(0/0911,0/1998,0/5305)	c_{gk}

According to Figure (7), the capability indices based on tolerance range in the triangular fuzzy number are lower than 1.33. Then the capability indices are week.

3.1.2. Device capability indexes calulations based on process range using triangular fuzzy numbers

Where variance is known as an obvious number, capability indexes are computed using triangular fuzzy numbers, they are presented in Table (5). Results of capability indexes based on process range using triangular fuzzy numbers are shown in Figure (8).

Table 5
Device capability indexes based on process range using triangular fuzzy numbers

capability indexes	
(0/082,0/1529,0/3581)	c_g
(0/0478,0/1129,0/3139)	c_{gk}

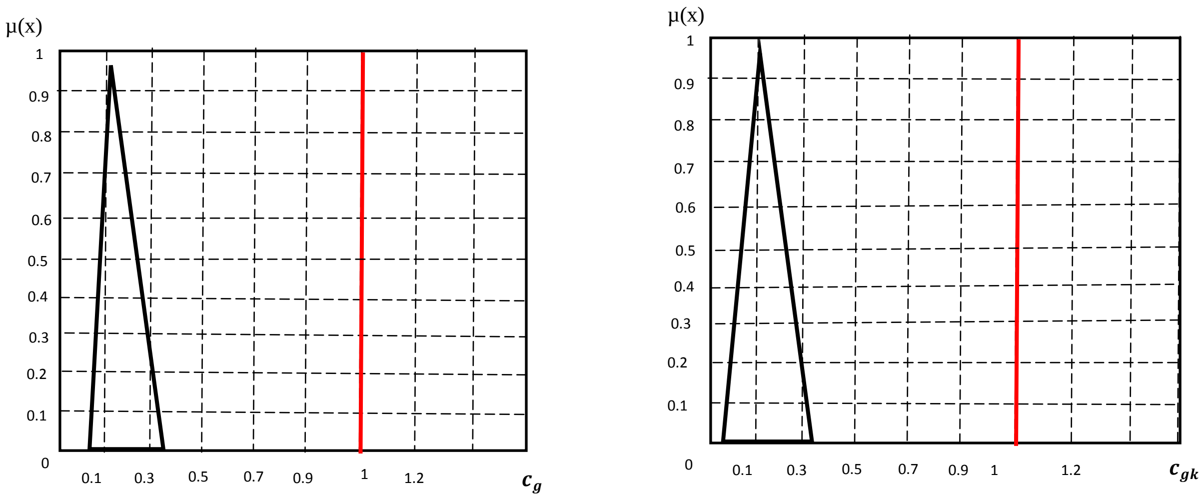

Fig.8. Capability indexes based on process range using triangular fuzzy numbers

According to Figure (8), the capability indices based on process range in the triangular fuzzy number are lower than 1. Then the capability indices are week.

3.2. Industrial data in form of trapezoidal fuzzy numbers

Table (6) represents trapezoidal fuzzy data. Parameters required for calculating device capability index are provided in Table (7).

Table 6
Trapezoidal fuzzy numbers.

NO.	x_1	x_2	x_3	x_4
1	50/006,50/007,50/008,50/009	50/004,50/005,50/006,50/007	50/007,50/008,50/009,50/010	50/006,50/007,50/008,50/009
2	50/007,50/008,50/009,50/010	50/008,50/009,50/010,50/011	50/003,50/004,50/005,50/006	50/004,50/005,50/006,50/007
3	50/006,50/007,50/008,50/009	50/009,50/010,50/011,50/012	50/004,50/005,50/006,50/007	50/007,50/008,50/009,50/010
4	50/006,50/007,50/008,50/009	50/005,50/006,50/007,50/008	50/008,50/009,50/010,50/011	50/008,50/009,50/010,50/011
5	50/004,50/005,50/006,50/007	50/004,50/005,50/006,50/007	50/006,50/007,50/008,50/009	50/004,50/005,50/006,50/007
6	50/006,50/007,50/008,50/009	50/008,50/009,50/010,50/011	50/004,50/005,50/006,50/007	50/005,50/006,50/007,50/008
7	50/005,50/006,50/007,50/008	50/005,50/006,50/007,50/008	50/005,50/006,50/007,50/008	50/007,50/008,50/009,50/010
8	50/004,50/005,50/006,50/007	50/005,50/006,50/007,50/008	50/004,50/005,50/006,50/007	50/006,50/007,50/008,50/009
9	50/004,50/005,50/006,50/007	50/007,50/008,50/009,50/010	50/004,50/005,50/006,50/007	50/007,50/008,50/009,50/010
10	50/004,50/005,50/006,50/007	50/007,50/008,50/009,50/010	50/005,50/006,50/007,50/008	50/003,50/004,50/005,50/006
11	50/003,50/004,50/005,50/006	50/006,50/007,50/008,50/009	50/006,50/007,50/008,50/009	50/008,50/009,50/010,50/011

12	50/004,50/005,50/006,50/007	50/001,50/002,50/003,50/004	50/005,50/006,50/007,50/008	50/004,50/005,50/006,50/007
13	50/005,50/006,50/007,50/008	50/005,50/006,50/007,50/008	50/004,50/005,50/006,50/007	50/004,50/005,50/006,50/007
14	50/007,50/008,50/009,50/010	50/006,50/007,50/008,50/009	50/005,50/006,50/007,50/008	50/007,50/008,50/009,50/010
15	50/006,50/007,50/008,50/009	50/006,50/007,50/008,50/009	50/004,50/005,50/006,50/007	50/008,50/009,50/010,50/011
16	50/004,50/005,50/006,50/007	50/003,50/004,50/005,50/006	50/005,50/006,50/007,50/008	50/005,50/006,50/007,50/008
17	50/004,50/005,50/006,50/007	50/005,50/006,50/007,50/008	50/009,50/010,50/011,50/012	50/005,50/006,50/007,50/008
18	50/005,50/006,50/007,50/008	50/004,50/005,50/006,50/007	50/006,50/007,50/008,50/009	50/005,50/006,50/007,50/008
18	50/002,50/003,50/004,50/005	50/006,50/007,50/008,50/009	50/005,50/006,50/007,50/008	50/006,50/007,50/008,50/009
20	50/002,50/003,50/004,50/005	50/005,50/006,50/007,50/008	50/004,50/005,50/006,50/007	50/004,50/005,50/006,50/007
21	50/002,50/003,50/004,50/005	50/003,50/004,50/005,50/006	50/005,50/006,50/007,50/008	50/004,50/005,50/006,50/007
22	50/003,50/004,50/005,50/006	50/006,50/007,50/008,50/009	50/006,50/007,50/008,50/009	50/003,50/004,50/005,50/006
23	50/005,50/006,50/007,50/008	50/010,50/011,50/012,50/013	50/005,50/006,50/007,50/008	50/005,50/006,50/007,50/008
24	50/005,50/006,50/007,50/008	50/008,50/009,50/010,50/011	50/005,50/006,50/007,50/008	50/003,50/004,50/005,50/006
25	50/007,50/008,50/009,50/010	50/004,50/005,50/006,50/007	50/007,50/008,50/009,50/010	50/006,50/007,50/008,50/009

Table 7
Parameters required for solving industrial problem with trapezoidal fuzzy numbers

parameters	
50/007,50/008,50/009,50/010	$\bar{X}=(\bar{X}_1,\bar{X}_2,\bar{X}_3,\bar{X}_4)$
(0/0015,0/0016,0/0017,0/0018)	VAR=(VAR,VAR,VAR_3,VAR_4)
(0/010,0/011,0/012,0/013)	T=(T_1,T_2,T_3,T_4)

3.2.1. Device capability indexes calulations based on tolerance range using trapezoidal fuzzy numbers

Table (8) shows device capability indexes based on tolerance range in industries using trapezoidal fuzzy numbers. Results of capability indexes based on tolerance range using trapezoidal fuzzy numbers are shown in Figure (9).

Table 8
Industrial device capability indexes based on tolerance range using trapezoidal fuzzy numbers

capability indexes	
(0/1852,0/2292,0/3333,0/5417)	c_g
(0/1111,0/1667,0/2778,0/5)	c_{gk}

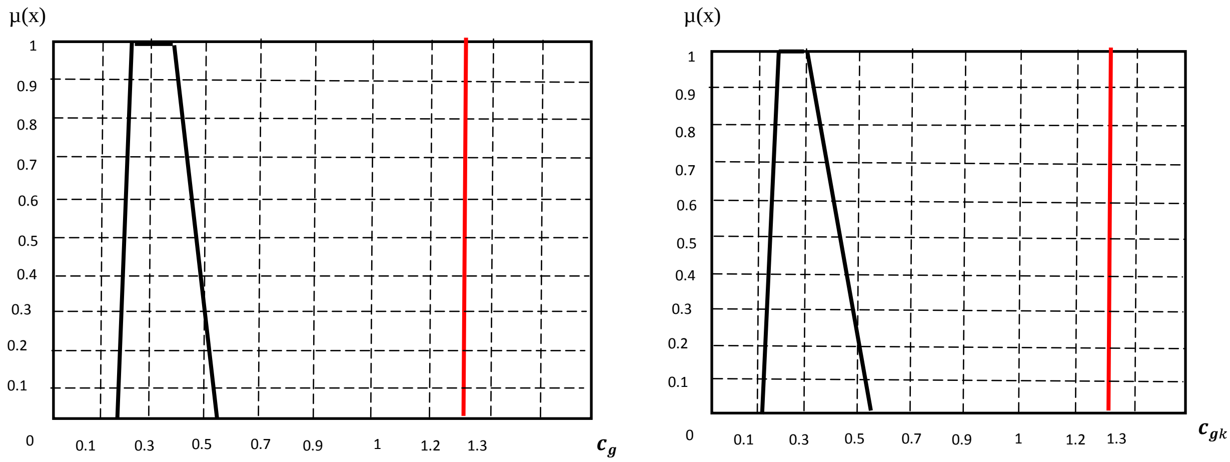

Figure (9) capability indexes based on tolerance range using trapezoidal fuzzy numbers

According to Figure (9), the capability indices based on tolerance range in the trapezoidal fuzzy number are lower than 1.3. Then the capability indices are week.

3.2.2. Device capability indexes calulations based on process range using trapezoidal fuzzy numbers

Table (9) presents device capability indexes based on process range in industries using trapezoidal fuzzy numbers. Results of capability indexes based on process range using trapezoidal fuzzy numbers are shown in Figure (10).

Table 9
Industrial device capability indexes based on process range using trapezoidal fuzzy numbers.

capability indexes	
(0/125,0/15,0/2125,0/3375)	c_g
(0/0509,0/0875,0/1569,0/2958)	c_{gk}

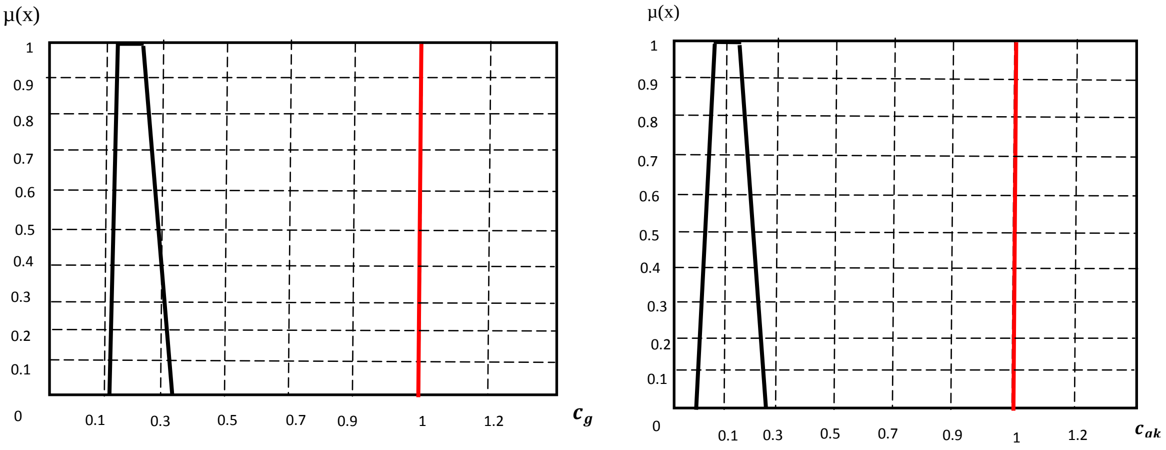

Fig. 10. Capability indexes based on process range using trapezoidal fuzzy numbers

According to Figure (10), the capability indices based on process range in the trapezoidal fuzzy number are lower than 1. Then the capability indices are week.

3.3. Calculating device capability indexes in certain environment

Device capability indexes in certain environment are calculated using assumed data and are avaliable in Tables (10) and (11), according to process and tolerance range.

Table10
Device capability indexes based on tolerance range using certain data.

capability indexes	
0/2394	c_g
0/2374	c_{gk}

Table 11
Device capability indexes based on process range using certain data

capability indexes	
0/1526	c_g
0/1506	c_{gk}

3.4. Statistical analysis of results gained from measurement system

According to statistic, T test is used to examine equality of means in two independent populations. We indicate average of first population as μ_1 , whereas μ_2 is used to indicate average of second population. Thus, hypothesis testing is accomplished from the following:

$H_0 : \mu_1 = \mu 2$
$H_1 : \mu_1 \neq \mu 2$

Base on the research hypothesis: measurement system analysis (MSA) experiences more sensitivity and efficiency in fuzzy environment rather than certain area, Following assumptions (hypothesis) are defined:
H_0: Sensitivity and efficiency of measurement system analysis are equal in both fuzzy and certain environment.

H_1: Sensitivity and efficiency of measurement system analysis are not equal (are different) in fuzzy and certain environment.
Here, hypothesis testing for comparing two population means in case of independency (independent T test) is used. Further, observations gained from two populations in fuzzy and certain environment are analyzed statistically. This test helps to investigate equality of two methods as well as introducing better method using confidence interval at confidence level 95%. Table (12) shows results obtained from independent T test.

3.4.1. Independent T test

The analysis is investigated in two cases which is shown in Table (12). According to Sig in both cases (where Sig is lower than 0.05 which is considered as test error, null hypothesis ($\mu 1 = \mu 2$) would be rejected), rejection of null

hypothesis indicating equality of efficiency and sensitivity in two methods is occurred. Therefore, it can be concluded that these two methods are not equivalent.
In Table 12, Sig, is test error and df, is degree of freedom. In our study, df is calculated by equation (18). Where, n is summation of certain and fuzzy data and t is the test statistics.

$$df = n - 2 \quad (19)$$

Table 12
Independent *T* test

Index	Independent T test				
	t-test for Equality of Means				
	t	df	Sig	95% Confidence Interval of the Difference	
				Lower	Upper
C_g	0.358	198	0.020	-0.0243874	-0.0168874
C_{gk}	2.007	198	0.046	-0.0383658	-0.0003342

Furthermore, results obtained from confidence interval lead us to conclude that fuzzy method (second population) applies more sensitivity and efficiency rather than certain method (first population). It is somehow connected with existence of negative values in upper and lower levels.

4. Conclusion, Discussion and Future Work

This paper develops a fuzzy approach for decision making about device capability indexes (C_{gk} , C_g). MATLAB software is used to solve the model and results suggested that presented model is highly sensitive in fuzzy environment rather than crisp environment. Our results in fuzzy and crisp environment show that the model runs properly. In other words, proximity of acceptable device capability levels and results of calculations with certain data are able to certify device capability and results of calculations regarding fuzzy environment reject related divice capability. In comparison to certain environment, there are deviations that suggest high sensitivity and flexibility in fuzzy environment.
On the other hand, the deviations in the trapezoidal fuzzy environment are greater than the triangular fuzzy environment.Table 11, presents results in certain environment, Tables 12 and 13 show results in fuzzy environment in two conditions: process and tolerance.
As shown in Figure (12) and (13), trapezoidal environment indicates more sensitivity rather than certain area and triangular fuzzy environment.
It proves that trapezoidal environment is highly sensitive rather than certain area. Moreover, for further researches, fuzzy mod, fuzzy mean, fuzzy principals and fuzzy Alfa-cut can be introduced as methods to develop device capability indexes (C_{gk} , C_g).

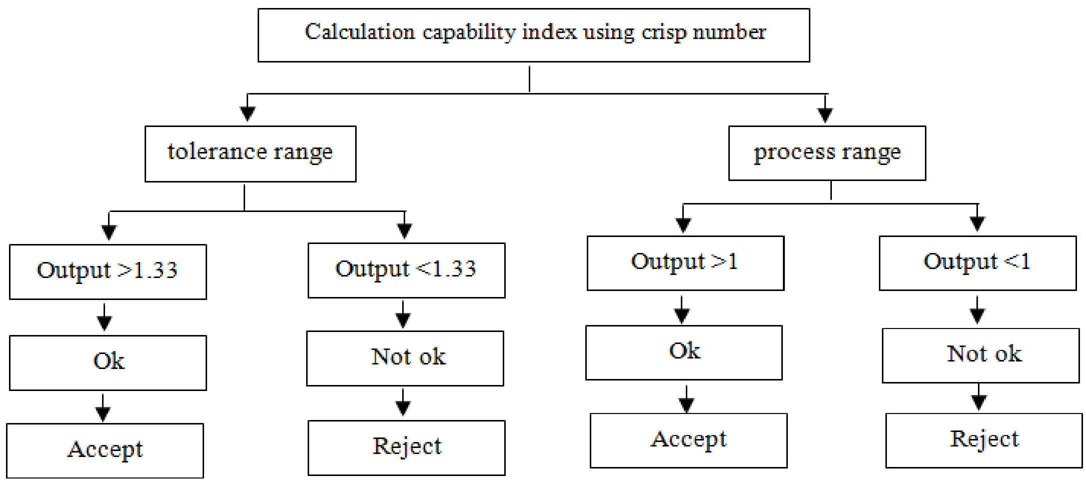

Fig. 11. Analysis of the results Calculation capability index using crisp number

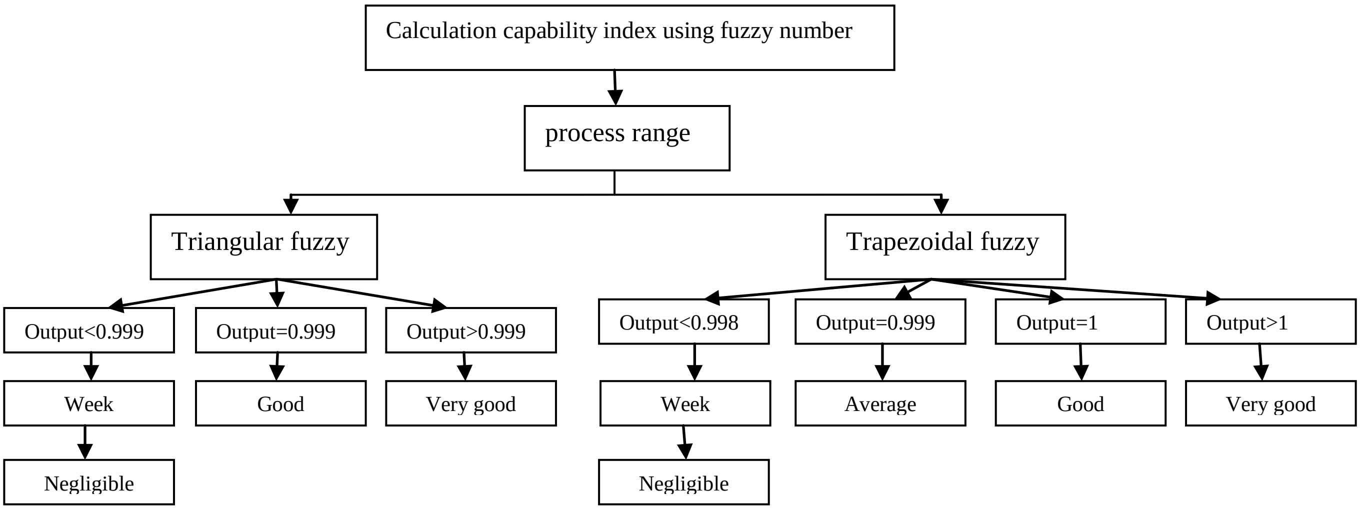

Fig. 12. Analysis of the results Calculation capability index using fuzzy number (process range)

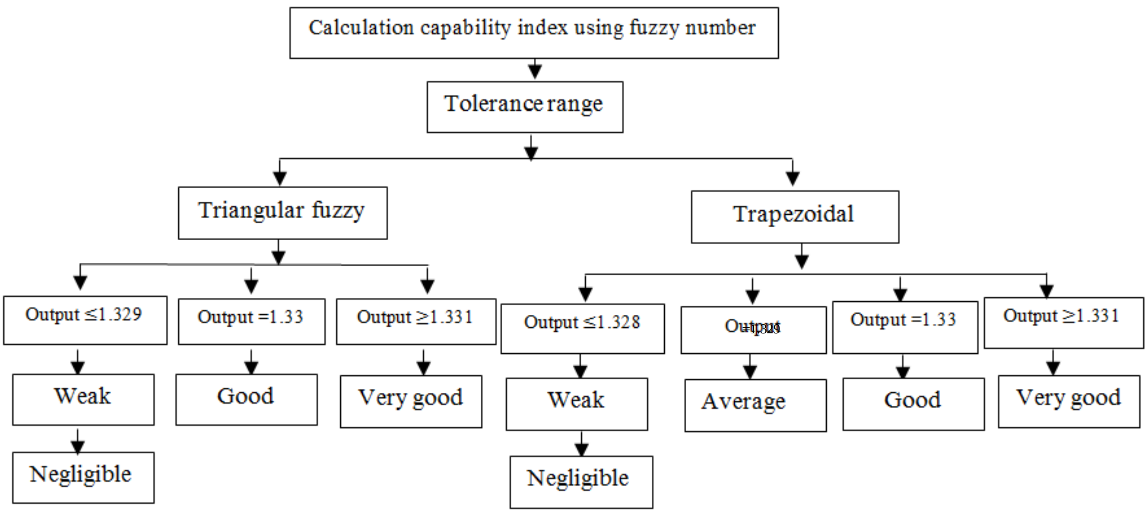

Fig. 13. Analysis of the results Calculation capability index using fuzzy number (Tolerance range)

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