

# A New Mathematical Model for the Green Vehicle Routing Problem by Considering a Bi-Fuel Mixed Vehicle Fleet

Neda Manavizadeh<sup>a,\*</sup>, Hamed Farrokhi-Asl<sup>b</sup>, Stanley Frederick W.T. Lim<sup>d</sup>

<sup>a</sup> Department of Industrial Engineering, KHATAM University, Tehran, Iran

<sup>b</sup> School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

<sup>c</sup> Institute for Manufacturing, University of Cambridge, Cambridge, United Kingdom

<sup>d</sup> W.P. Carey School of Business, Arizona State University, Tempe, United States

Received 21 June 2019; Revised 09 December 2019; Accepted 23 February 2020

## Abstract

This paper formulates a mathematical model for the Green Vehicle Routing Problem (GVRP), incorporating bi-fuel (natural gas and gasoline) pickup trucks in a mixed vehicle fleet. The objective is to minimize overall costs relating to service (earliness and tardiness), transportation (fixed, variable and fuel), and carbon emissions. To reflect a real-world situation, the study considers: (1) a comprehensive fuel consumption function with a soft time window, and (2) an *en-route* fuel refueling option to eliminate the constraint of driving range. A linear set of valid inequalities for computing fuel consumption were introduced. In order to validate the presented model, first, the model is solved for an illustrative example. Then each component of cost objective function is considered separately so as to investigate the effects of each part on the obtained solutions and the importance of vehicles speed on transportation strategies. Computational analysis shows that, despite the limitation of an appropriate service infrastructure, the proposed model demonstrated an average reduction of 44%, 6% and 5% in carbon emission costs, total distribution costs, and transportation costs respectively. Moreover, the study found paradoxical effects of average speed, suggesting the need to manage trade-offs: while higher speeds reduced service costs, they increased carbon emission costs. In the next stage, some experiments modified from the literature are solved. According to these experiments, in all instances greater objective function values for Gasoline vehicles are gained. The difference in the carbon emission objective is also significant, with an average of 44.23% increase. Finally, managerial and institutional implications are discussed.

**Keywords:** Green vehicle routing; Carbon emission; Bi-fuel light truck; Soft time window; Green logistics

## 1. Introduction

Transportation has various hazardous effects on the environment (Koç and Karaoglan, 2016), including toxic effects on ecosystems, noise pollution, acidification, depletion of the ozone layer, and the greenhouse effect (Knörr, 2011). Globally, road transportation contributes an estimated 21% of the overall carbon dioxide (CO<sub>2</sub>) emissions (Jabali, Woensel, and de Kok 2012). Consequently, institutions are implementing freight regulations and carbon emission policies in order to regulate vehicle footprints and limit emissions by companies (Quak and Dekoster, 2007; Bynum et al. 2018). In response, firms are pressured to implement “green logistics” initiatives in order to reduce their carbon footprint. A commonly adopted approach in the transportation industry, which utilizes analytical modeling to optimize vehicle routing and scheduling, in order to reduce the overall travelled distance and the corresponding carbon emissions.

The study of vehicle routing problems (VRP) has evolved overtime since its inception in 1959 (Dantzig and Ramser,

1959), in an attempt to capture these emerging trends. This triggered the modifications to traditional VRPs, where environmental constraints were incorporated into the planning and management of vehicles (Rabbani et al., 2016; Rabbani et al. 2018; Farrokhi-Asl et al. 2018). These modifications have, amongst others, produced a new variant of VRP known as the Green VRP (GVRP) (Lin et al, 2014), with the objective of minimizing fuel consumption or CO<sub>2</sub> emissions and operating costs, concomitantly fulfilling customer demanded service levels (Bektaş and Laporte, 2011).

While the extant literature has focused exclusively either on fossil fuel vehicles or alternative fuel vehicles (AFVs) in GVRP modeling, the benefits of hybridization (i.e. bi-fuel) to address the shortcoming of higher carbon emission intensity (Association, 2004) and price associated with fossil fuel or the lack of an appropriate service infrastructure and the constraint in driving range, associated with alternative fuel (e.g. compressed natural gas [CNG]), have promoted increasing adoptions in the transportation industry. Companies are gradually equipping their production lines with bi-fuel pickup trucks

\*Corresponding author Email address: n.manavi@khatam.ac.ir

(e.g. Huang et al. 2016). Of significance, bi-fuel trucks are capable of efficiently distributing over a wider geographical area and have the flexibility to switch fuel type *en-route*.

Despite the increasing relevance, studies addressing bi-fuel (e.g. fossil fuel and CNG) mixed vehicle fleets in VRPs are limited. Most of these studies, although have consistently reported reductions in emissions (e.g. Palmer, 2007; Maden et al., 2010), they only address specific issues through incremental adjustments to the GVRP; hence, they do not generally fit well with industrial applications, resulting in limited use for managers. Our retrospective analysis of the literature revealed that the body of knowledge has advanced in two directions: (1) GVRP with an *en-route* refueling option, using a constant rate of fuel consumption function, and (2) GVRP with a comprehensive fuel consumption function without considering an *en-route* refueling option. For example, Omidvar and Tavakkoli-Moghaddam (2012) implemented an *en-route* refueling model for alternative fuel vehicles, accounting for traffic conditions; Schneider et al., (2014) formulated a recharging problem for electronic vehicles by considering time windows restriction; and Erdoğan and Miller-Hooks (2012) included refueling options for vehicles either by fossil fuel or alternative fuel along the routes. These studies make a simplistic assumption of constant rates of fuel consumption. To address this limitation, several studies developed variants of a comprehensive fuel consumption function to better reflect realities, and further incorporate time windows, freight intensity, distance and speed (Palmer, 2007; Maden et al., 2010; Pradenas et al., 2013; Bektaş and Laporte, 2011). However, these studies do not consider an *en-route* refueling option.

Finally, while Zhang and Wang (2013) considered a mixed fleet VRP (MFVRP) to reduce fuel consumption, they modeled the problem as comprising open routes without obligation to return to depots upon completion of delivery. Because they employed an approximation method to solve the generated instances, there was no baseline to compare the performance of their proposed algorithm.

Consolidating these developments in GVRP, our study addresses the knowledge gap in modeling bi-fuel mixed vehicle fleets, by considering both refuel option and a comprehensive fuel consumption function. Specifically, we formulated a mixed integer linear model to incorporate a mixed fleet of bi-fuel vehicles with changeable fuel types along each arc of a route, and introduced a consumption rate function for each fuel type. The function considers fuel type specification, pure vehicle weight, freight weight, traveled distance, and average speed of a vehicle. These parameters are then fed into two valid inequalities, in order to accumulate and capture fuel consumption along every arc of each route. Our objective function consisted of three cost terms: transportation (Azadeh and Farrokhi-Asl 2017), service (Rabbani et al. 2015), and carbon emissions (Toro et al. 2017).

Our study introduced the concept of utilizing bi-fuel vehicles along the routes with the option of refueling. In

so doing, we proposed a formulation of the GVRP with a bi-fuel mixed vehicle fleet, incorporating linear equations to calculate the fuel consumption of each fuel type. Our computational analysis shows that bi-fuel vehicles contribute to a 44% reduction in carbon emission costs, 6% reduction in the total costs of distribution, and 5% reduction in transportation costs. Our findings further suggest a trade-off between carbon emission and service costs. For example, if companies prioritize on service time, their vehicle fleet size would have to increase in order to meet their customers' expected time window.

The remainder of this paper is organized as follows: Section 2 reviews the literature. Section 3 defines the problem and formulates the mathematical model for a bi-fuel mixed vehicle fleet. Section 4 provides an application illustration building on the formulated model. Computational analysis is carried out in Section 5 and Section 6 concludes.

## 2. Literature Review

In this section, we provide a brief review of the literature on GVRP<sup>1</sup>, focusing on the fuel types to classify vehicles and introducing the theoretical foundations, in terms of time windows, function objectives and mixed fleet VRP (MFVRP), which this study has built upon for the proposed model formulation.

### 2.1 Review of vehicle types used in GRVP based on fuel type

A review of the extant literature revealed three types of vehicles based on their fuel type: fossil fuel (e.g. gasoline) (Bektaş and Laporte 2011; Demir, Bektaş, and Laporte 2012, 2014), alternative fuel (e.g. CNG, Liquefied Natural Gas [LNG], electronic) (Erdoğan and Miller-Hooks 2012; Yavuz et al. 2015; Azadeh et al. 2017), and double fuel (e.g. hybrid, bi-fuel, dual-fuel) (Salimifard and Raeesi 2014). While the literature appears to be dominated by studies using fossil fuel and alternative fuel vehicles, studies considering double fuel (or bi-fuel hereafter) are limited (Salimifard and Raeesi 2014).

On fossil fuel type vehicles, Palmer (2007) formulated a GVRP with time window and variant speed and reported a 5% reduction in freight carbon emissions. Maden et al. (2010) considered Time Dependent GVRP with a 7% reduction in carbon emissions. Pradenas et al. (2013) studied Backhaul VRPTW using a scatter search and cooperative game approach claiming a 30% reduction in emissions cost. Jabali, Woensel, and de Kok (2012) modeled a VRP considering traffic conditions and compared two cases of the Emissions-based Time-Dependent Vehicle Routing Problem (E-TDVRP). By considering only CO<sub>2</sub> emissions and traveling time (as a single objective function), the study demonstrated that an 11.4% and 3.2% reduction in emissions and cost,

<sup>1</sup> Reader is referred to Lin et al. (2014) for an extensive review on the classification between traditional and green vehicle routing problem.

respectively. Huang et al. (2012) proposed a linear integer programming model for GVRP with simultaneous pickups and deliveries (GVRPSPD) to reduce the carbon emission costs. Focusing on the fuel consumption function, Kara, Kara, and Tetis (2007) incorporated the weight of vehicle along with the traveled distance in the function. Extending this study, Xiao et al. (2012) added freight weight while Bektaş and Laporte (2011) combined the average speed of a vehicle in the VRPTW. Although, Barth, Younglove, and Scora (2005) employed a simplified function to investigate vehicle drive-train efficiency, they developed a comprehensive model for the fuel consumption rate model and validated it through numerical results. As such, we adopted their fuel consumption function in this study.

Turning to alternative fuel type vehicles (AFVs), Erdoğan and Miller-Hooks (2012) proposed the option of refueling along a route. Schneider et al. (2014) presented the *en-route* recharging problem for electronic vehicles by considering time windows restriction (E-VRPTW). Further, Omidvar and Tavakkoli-Moghaddam (2012) developed a GVRP with *en-route* refueling and included traffic conditions to reflect real-world situations. However, this type of solution often faced implementation issues in practice due to infrastructural constraint (Lowe 2016). Therefore, relying solely on AFVs is infeasible.

Lastly on bi-fuel type vehicles, Salimifard and Raeesi (2014) developed a new variant of GVRP by considering CO<sub>2</sub> emissions and costs stemming from a bi-fuel vehicle fleet. According to our review, this appears to be the only paper that incorporates bi-fuel vehicles in the formulation of GVRP. Extending this body of literature, we adapt Salimifard and Raeesi's (2014) approach to formulate fuel consumption costs for single type vehicles, by incorporating the characteristics of bi-fuel vehicle types in the fuel consumption function, and consider heterogeneous vehicle fleets.

Based upon these developments in the GVRP literature, it is evident that there is a clear lack of study on bi-fuel mixed vehicle fleets, incorporating a refueling option along a route, and a realistic fuel consumption function. In order to better reflect the real-world complexities, additional parameters such as time windows are essential to be included in the model, and this is where we turn to next.

## 2.2 Review of time windows, function objectives and mixed fleet VRP

**Time windows.** One of the most important factors in home delivery is the customer's preferred time slot for the reception of goods, also known as service time criteria (Russell, 1977; Solomon, 1987; De Grancy and Reimann, 2015). Solomon (1987) introduced two types of time windows: soft and hard. In the case of a hard time window, vehicles must arrive for each customer within a determined time window. Whereas, in a soft time window situation, the violation of determined time is acceptable, but will incur a penalty cost (Min, 1991). Due to constraint enforcement of hard time windows to a solution

space, soft time windows are more practical. Several studies exist that have used soft time windows. For example, Dell'Amico et al. (2007) developed a VRP with time windows in which various types of vehicles, with a deferent capacity, were utilized for distribution. The composition of vehicles and the routing of a fleet of heterogeneous vehicles were considered simultaneously in order to serve a given set of customers. Sexton and Choi (1986) proposed a single-VRP combining pick-up and delivery with soft time windows to reduce the cost of network flow and service to the customer. Min (1991) developed a multi-objective model for the routing problem with soft time windows in the case of a public library book distribution. Finally, Guerriero et al. (2014) introduced a model for unmanned aerial VRP with soft time window constraints. To implement these soft time windows, the determination of penalty costs for missing customers' preferred time slots were required.

**Function objectives.** We specified three components in the objective function: transportation, service and carbon emission costs. Transportation costs comprise two elements: fixed and variable costs (Azadeh and Farrokhi-Asl 2017). Variable cost is directly proportional with the length of routes. By contrast, fixed cost is incurred when a vehicle departs from the depot for operation. Each vehicle has its own specific fixed cost independent of the length and traveling time of its corresponding route. The service component in the objective function is directly related to customer satisfaction (Rabbani et al. 2015). Satisfaction levels increased when the customer was serviced within the desired time window. Lastly, the carbon emissions component in the objective function specified the "greenness" of the transportation activities and was in direct association with the mileage traveled (Toro et al. 2017). The goal was to capture the trade-off between operational and environmental considerations in the objective function.

**Mixed fleet VRP (MFVRP).** Daneshzand (2011) surveyed three types of MFVRP. The first type employed identical variable costs, coupled with an unlimited number of vehicles (e.g. Golden et al., 1984). The second type used different variable costs and an unlimited number of vehicles (e.g. Salhi, et al., 1992). The third type utilized different variable costs and a limited number of vehicles (e.g. Taillard, 1999). Dell'Amico et al. (2007) further studied the problem associated with vehicles of heterogeneous capacities and traveled distance costs, while Zhang and Wang (2013) considered MFVRP with various implementation costs in order to reduce fuel consumption.

Distinct from the aforementioned classical operational research models, this study simultaneously addressed two critical operational considerations in vehicle routing, namely – time windows and a mixed vehicle fleet within the context of GVRP.

## 3. Problem Formulation

In value chain definition, logistics is considered as one of three value-added goods and services with a direct impact

on operational costs and firm profits (Christopher 2005). Green logistics compliment efficiency and effectiveness in total supply chain procedures (Wilson and Gilligan, 2012). For the first time, we consider the impact of a bi-fuel mixed vehicle fleet, on three operational costs: transportation, service, and carbon emissions. Moreover, we consider the option of refueling along the route, and utilize a comprehensive fuel-type dependent consumption function on any route. The function depends on the weight of freight and vehicle, average speed, and traveled distance. In addition to capturing fuel types for the purpose of estimating fuel consumption, we considered vehicle type and operational efficiency. We further included assumptions, both, that have been previously specified in the literature as well as those that mimic real-world situations. They were elaborated in the following.

- Refueling along a route was permitted without upper bound limitation;
- Vehicles had heterogeneous tank and freight capacity as well as pure weight;
- Vehicles could use fossil and CNG fuels and there was a possibility of fuel swapping;
- Closed VRP model, i.e. every vehicle initiated from the depot and made a return journey to the depot;
- Vehicles were bi-fuel and only CNG refueling stations were considered;
- At the start of each route, the vehicle had a full tank of both CNG and fossil fuels;
- At each gas station, vehicles had the option to refuel any or both fuel types; and
- Each vehicle was permitted to visit each customer only once.

Let  $V$  be a set of vertices ( $V = I \cup \Omega \cup \{v_0, v_{N+1}\}$ ), where  $I$  and  $\Omega$  is a set of customers ( $I = \{v_1, v_2, \dots, v_N\}$ ) and gas stations ( $\Omega = \{\omega_1, \omega_2, \dots, |\Omega|\}$ ), respectively. Depot is denoted by  $v_0$  and  $v_{N+1}$  simultaneously. A complete directed graph  $G = (V, A)$ , with the set of nodes  $V$  and arcs  $A = \{(v_i, v_j): v_i, v_j \in V, j \neq i\}$  were defined. The set of fuel types were defined by  $F = \{1, 2\}$ , in which the number '1' and '2' indicated CNG and gasoline fuels, respectively. Finally, let  $T = \{1 \cdot 2 \cdot \dots \cdot |T|\}$  denote the set of heterogeneous trucks. The parameters and variables were defined as follows:

**Indices:**

- $i, j$  index of all nodes in the network
- $t$  index of trucks
- $f$  index of fuel type

**Parameters:**

- $D_i$  Demand of customer  $i$  (kg)
- $\zeta_i$  Service time at location  $i$  (seconds)
- $\gamma_i^+$  Late arrival penalty cost at location  $i$  (seconds)
- $\gamma_i^-$  Early arrival penalty cost at location  $i$  (seconds)
- $C^t$  Maximum freight weight of truck  $t$  (kg)
- $W^t$  Vehicle's  $t$  pure weight
- $Q^{tf}$  Tank capacity of fuel  $f$  for vehicle  $t$
- $d_{ij}$  Traveled distance from  $i$  to  $j$  (m)
- $\kappa^f$  Fuel cost of  $f$  type (\$)
- $\pi^+$  Late arrival penalty cost (seconds)
- $\pi^-$  Early arrival penalty cost (seconds)
- $p^f$  Emission cost of fuel type  $f$  (\$)
- $\delta_{ij}$  Travelling time from  $i$  to  $j$  nodes (seconds)
- $\psi^t$  Kilometer travelling cost of vehicle  $t$  (\$)
- $\Psi^t$  Fixed acquisition cost of vehicle  $t$  (\$)

**Decision variables:**

- $x_{ij}^{tf}$  Binary variable; equal to 1 if truck type  $t$  with fuel type  $f$  travels from customer  $i$  to customer  $j$  and equal to 0 otherwise
- $\tau_j^t$  Arrival time of vehicle  $t$  at node  $j$
- $e_j^+$  The difference of arrival time at node  $j$  with upper time of customers  $j$ 's time window
- $e_j^-$  The difference of arrival time at node  $j$  with lower time of customers  $j$ 's time window
- $u_j^t$  Vehicle  $t$ 's freight level at node  $j$
- $r_{ij}^{tf}$  Required fuel type  $f$  for traversing arc  $(i, j)$  by vehicle  $t$
- $y_j^{tf}$  Vehicle  $t$ 's fuel level of type  $f$  at node  $j$

With the bi-fuel vehicles, we considered a mixed vehicle fleet VRP with soft time windows, corresponding to customers' expected delivery time slots. Our proposed model relied on the recharging option for electronic vehicles on a heterogeneous and wide network. Accordingly, the distance between nodes was long. In addition, we adopted the fuel consumption function from Barth, Younglove, and Scora (2005). Based upon the defined sets, parameters, and variables, the proposed model was specified as follows:

$$\begin{aligned}
 \text{Min} \quad & \sum_{j \in V_{N+1}} \sum_{t \in T} \sum_{f \in F} \Psi^t \cdot x_{0j}^{tf} + \sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{t \in T} \sum_{f \in F} \psi^t \cdot d_{ij} \cdot x_{ij}^{tf} + \sum_{j \in I} (\pi^+ \cdot e_j^+ + \pi^- \cdot e_j^-) \\
 & + \sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{t \in T} \sum_{f \in F} (\kappa^f + p^f) \cdot r_{ij}^{tf} \cdot x_{ij}^{tf}
 \end{aligned} \tag{1}$$

s. t

$$\sum_{j \in I, j \neq i} \sum_{t \in T} \sum_{f \in F} x_{ij}^{tf} = 1 \quad \forall i \in V_0 \quad (2)$$

$$\sum_{t \in T} \sum_{f \in F} \sum_{j \in V \setminus \{v_0, v_{N+1}\}} x_{0j}^{tf} - \sum_{t \in T} \sum_{f \in F} \sum_{i \in V \setminus \{v_0, v_{N+1}\}} x_{iN+1}^{tf} = 0 \quad (3)$$

$$\sum_{i \in V_{N+1}, i \neq j} \sum_{f \in F} x_{ji}^{tf} - \sum_{i \in V_0, i \neq j} \sum_{f \in F} x_{ij}^{tf} = 0, \quad \forall j \in V \setminus \{v_0, v_{N+1}\}, \forall t \in T \quad (4)$$

$$\sum_{j \in V \setminus \{v_0\}} \sum_{f \in F} x_{0j}^{tf} \leq 1 \quad \forall t \in T \quad (5)$$

$$r_{ij}^{tf} = (\varepsilon^f \cdot \eta^f)^{-1} \cdot (\alpha_{ij}(u_i^t + W^t)d_{ij} + \beta^t v^2 d_{ij}), \quad \forall i, j \in V, i \neq j, \forall t \in T, \forall f \in F \quad (6)$$

$$y_j^{t1} \leq y_i^{t1} - r_{ij}^{t1} \cdot x_{ij}^{t1} + Q^{t1} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1} \setminus \Omega, i \neq j \quad (7)$$

$$y_j^{t1} \geq y_i^{t1} - r_{ij}^{t1} \cdot x_{ij}^{t1} - Q^{t1} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1} \setminus \Omega, i \neq j \quad (8)$$

$$y_j^{t2} \leq y_i^{t2} - r_{ij}^{t2} \cdot x_{ij}^{t2} + Q^{t2} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1}, i \neq j \quad (9)$$

$$y_j^{t2} \geq y_i^{t2} - r_{ij}^{t2} \cdot x_{ij}^{t2} - Q^{t2} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall t \in T, \forall i \in V_0, \quad \forall j \in V_{N+1}, i \neq j \quad (10)$$

$$y_i^{tf} - r_{ij}^{tf} \geq 0 \quad \forall i \in V_0, \forall j \in V_{N+1}, i \neq j, \forall t \in T, \forall f \in F \quad (11)$$

$$y_i^{tf} \leq Q^{tf} \cdot \sum_{j \in V_{N+1}, j \neq i} \sum_{f \in F} x_{ij}^{tf} \quad \forall i \in V_0, \forall t \in T, \forall f \in F \quad (12)$$

$$y_j^{t0} = Q^{t0} \cdot \sum_{i \in I} \sum_{f \in F} x_{ij}^{tf} \quad \forall j \in \Omega \cup \{v_0\}, \forall t \in T \quad (13)$$

$$0 \leq u_j^t \leq u_i^t - D_j \cdot \sum_{f \in F} x_{ij}^{tf} + C^t \cdot \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall i \in V_0, \forall j \in V_{N+1}, i \neq j, \forall t \in T, \forall f \in F \quad (14)$$

$$0 \leq u_i^t \leq C^t \cdot \sum_{j \in V_{N+1}, j \neq i} \sum_{f \in F} x_{ij}^{tf} \quad \forall i \in V_0, \forall t \in T \quad (15)$$

$$u_0^t = D_i \cdot \sum_{i \in I} \sum_{j \in I, j \neq i} \sum_{f \in F} x_{ij}^{tf} \quad \forall t \in T \quad (16)$$

$$0 \leq \tau_i^t + (\delta_{ij} + \zeta_i) \cdot \sum_{f \in F} x_{ij}^{tf} - T_{max} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right) \leq \tau_j^t \quad \forall i \in V_0, \forall j \in V_{N+1}, i \neq j, \forall t \in T \quad (17)$$

$$\tau_i^t + (\delta_{ij} + \zeta_i) \cdot \sum_{f \in F} x_{ij}^{tf} + T_{max} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right) \geq \tau_j^t, \quad \forall i \in V, \forall j \in V_{N+1}, i \neq j, \forall t \in T \quad (18)$$

$$\tau_{N+1}^t \leq T_{max} \quad \forall t \in T \quad (19)$$

$$0 \leq \tau_i^t \leq T_{max} \cdot \sum_{j \in V_{N+1}, j \neq i} \sum_{f \in F} x_{ij}^{tf} \quad \forall i \in V_0, \forall t \in T \quad (20)$$

$$e_i^- \geq \gamma_i^- - \sum_{t \in T} \tau_i^t \quad \forall i \in I \quad (21)$$

$$e_i^+ \leq \gamma_i^+ - \sum_{t \in T} \tau_i^t \quad \forall i \in I \quad (22)$$

$$x_{ij}^{tf} \in \{0,1\}, r_{ij}^{tf} \geq 0, \quad \forall i,j \in V, i \neq j, \forall t \in T, \forall f \in F \quad (23)$$

$$z_j^t \geq 0, \quad \forall t \in T, \forall j \in \Omega, e_j^-, e_j^+ \geq 0 \quad \forall j \in I$$

Objective function (1) comprises fixed and variable costs of vehicles, penalty costs associated with delivery times (i.e., early and late arrival), and fuel and carbon emission costs. Constraint (2) states that each customer should be visited exactly one time. Flow conservation constraints are defined by constraints (3) and (4), while constraint (5) enforces all vehicles leaving the depot, at most one time. Constraint (6) denotes the fuel consumption function along any traversed arc. This function depends on the vehicles' net and freight weight, average speed, and traveled distance. The CNG fuel level at each node except CNG stations was specified by constraints (7) and (8). Likewise, gasoline fuel levels at all nodes were covered by constraints (9) and (10). If a vehicle used the fuel type  $f$  for traversing any arc, the level of fuel type  $f$  must at least exceed the requirement, guaranteed by constraint (11). Further, fuel levels must be positive for those nodes

that a vehicle visits; constraint (12). Constraint (13) denotes CNG fuel levels after refueling at any station, i.e. the CNG tank must be full after refueling. Through constraint (14), the load level at each node for each vehicle is determined. Constraint (15) states that the load level at any unvisited node must be equal to zero. Hence, the load level at the depot must be equal to the sum of visited nodes' demand, operationalized by constraint (16). Constraints (17)-(20) show the time of arrival at each node. Finally, constraints (21) and (22) calculate the difference between the actual arrival time and given time window of each customer.

Because  $y_i^{tf} \forall i \in \Omega$  does not show CNG levels when arriving at a station, the following additional constraints should be added to model in (1)-(23):

$$z_j^t \leq y_i^{t0} - r_{ij}^{t0} \cdot x_{ij}^{t0} + Q^{t0} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall t \in T, \forall i \in V_0, \forall j \in \Omega, i \neq j \quad (24)$$

$$z_j^t \geq y_i^{t0} - r_{ij}^{t0} \cdot x_{ij}^{t0} - Q^{t0} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall t \in T, \forall i \in V_0, \forall j \in \Omega, i \neq j \quad (25)$$

$$z_i^t \leq Q^{t0} \cdot \sum_{j \in V_{N+1}, j \neq i} \sum_{f \in F} x_{ij}^{t0} \quad \forall i \in \Omega, \forall t \in T \quad (26)$$

where variable  $z_j^t, \forall t \in T, \forall j \in \Omega$ , denotes the CNG level while arriving at each station. Clearly, this proposed model can be easily extended to represent bi-fuel stations

by replacing constraints (7)-(10) with the following pair constraints:

$$y_j^{tf} \leq y_i^{tf} - r_{ij}^{tf} \cdot x_{ij}^{tf} + Q^{tf} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1}, i \neq j \quad (27)$$

$$y_j^{tf} \geq y_i^{tf} - r_{ij}^{tf} \cdot x_{ij}^{tf} - Q^{tf} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \quad \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1}, i \neq j \quad (28)$$

Expression (1), (7), (8), (9), and (10) are non-linear, since there exists a multiplication of two variables ( $r_{ij}^{tf} \cdot x_{ij}^{tf}$ ). Therefore, we rely on linearization of bilinear terms in the MINLP to transform these expressions to an MILP model, in order to be solved by the traditional branch-and-bound

algorithms. A function of two variables is bilinear if it is linear with respect to each of its variables. Based on this, we replaced constraint (6) with the following:

$$r_{ij}^{tf} \leq (\varepsilon^f \cdot \eta^f)^{-1} \cdot \left( \alpha_{ij} \cdot (u_i^t + W^t) \cdot d_{ij} + \beta^t \cdot \frac{(d_{ij})^3}{(\delta_{ij})^2} \right) + M \cdot (1 - x_{ij}^{tf}), \forall i \in V_0, j \in V_{N+1}, i \neq j, \forall t \in T, \forall f \in F \quad (29)$$

$$r_{ij}^{tf} \geq (\varepsilon^f \cdot \eta^f)^{-1} \cdot \left( \alpha_{ij} \cdot (u_i^t + W^t) \cdot d_{ij} + \beta^t \cdot \frac{(d_{ij})^3}{(\delta_{ij})^2} \right) - M \cdot (1 - x_{ij}^{tf}), \forall i \in V_0, j \in V_{N+1}, i \neq j, \forall t \in T, \forall f \in F \quad (30)$$

$$r_{ij}^{tf} \leq M \cdot x_{ij}^{tf}, \forall i \in V_0, j \in V_{N+1}, i \neq j, \forall t \in T, \forall f \in F \quad (31)$$

The variable  $r_{ij}^{tf}$  becomes positive if and only if the associated arc  $(i, j)$  is traversed by vehicle  $t$  by using fuel  $f$ .

Thereafter, we can replace the non-linear constraints with inequalities as shown:

$$\text{Min} \sum_{j \in V \setminus \{v_0\}} \sum_{t \in T} \sum_{f \in F} \psi^t \cdot x_{0j}^{tf} + \sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{t \in T} \sum_{f \in F} \psi^t \cdot d_{ij} \cdot x_{ij}^{tf} + \quad (32)$$

$$\sum_{j \in I} (\pi^+ \cdot e_j^+ + \pi^- \cdot e_j^-) + \sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{t \in T} \sum_{f \in F} (\kappa^f + p^f) \cdot r_{ij}^{tf}$$

$$y_j^{t1} \leq y_i^{t1} - r_{ij}^{t1} + Q^{t1} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1} \setminus \Omega, i \neq j \quad (33)$$

$$y_j^{t2} \leq y_i^{t2} - r_{ij}^{t2} + Q^{t2} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1}, i \neq j \quad (34)$$

$$y_j^{t1} \geq y_i^{t1} - r_{ij}^{t1} - Q^{t1} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1} \setminus \Omega, i \neq j \quad (35)$$

$$y_j^{t2} \geq y_i^{t2} - r_{ij}^{t2} - Q^{t2} \left( 1 - \sum_{f \in F} x_{ij}^{tf} \right), \forall t \in T, \forall i \in V_0, \forall j \in V_{N+1}, i \neq j \quad (36)$$

#### 4. An Illustrative Example

As previously highlighted, bi-fuel vehicles are suitable for heterogeneous and wide networks because of their extended durability of fuel consumption. In order to define the problem comprehensively, we devised an illustrative example considering two distinct strategies. In the example shown in Figure 1, we examined a bi-fuel truck (CNG and gasoline) with a fuel consumption function considering fixed speed, fuel type specification, freight weight, and traveled distance. The objective function minimizes the sum of costs associated with transportation, carbon emissions, and service time. The fuel tank capacity of CNG and fossil fuel were  $30 \text{ m}^3$  and 60 Liters, respectively. The average speed of the vehicle was 60 km per hour and three fueling stations were available in the region. The pure weight of the vehicle was 5800 kg and the other parameters were defined in Table 1 and 2. Distance between nodes in the network are shown in Table 1. In addition, Table 2 determines demand of each customer and time windows at each column. A

schematic network of the described case was depicted in Figure 1. It should be noted that the distance matrix was symmetric; that is, the distance from node  $i$  to node  $j$  was equal to the traveling distance between node  $j$  and  $i$ .

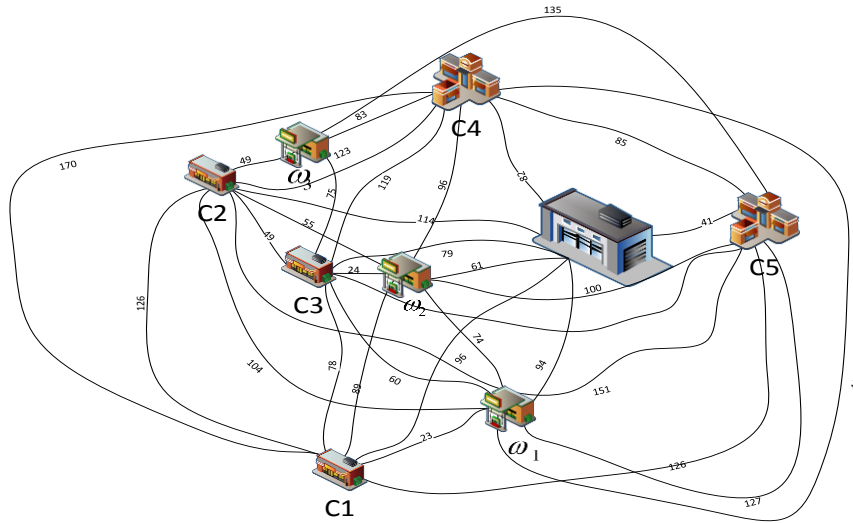


Fig. 1. The schematic network of illustrative example

Table 1  
Distance between nodes

depot	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$\omega_1$	$\omega_2$	$\omega_3$
	96	114	79	82	41	94	61	105
		124	78	170	126	23	89	151
			49	123	151	104	55	49
				119	118	60	24	75
					85	162	96	83
						127	100	135
							74	133
								64

Table 2  
Customer demand-related parameters

Customer	Demand	Earliest allowable time (seconds)	Latest allowable time (seconds)
$c_1$	150	2000	26000
$c_2$	300	1500	35000
$c_3$	150	500	25000
$c_4$	150	700	30000
$c_5$	300	3000	23000

Regarding the objective function components, we considered different fixed and identical variable costs for simplicity. The fuel consumption function  $\eta^f$  was the multiplication of engine and vehicle drive-train efficiencies; the engine efficiency using CNG and gasoline fuels were 0.39 and 0.41, respectively. Also, the vehicle drive-train efficiency was usually between 0.75 and 0.94 (Herein, we considered efficiency of 0.75). The

energy of one liter of gasoline was 8.8 kWh (Fuchs and Masoum, 2011), whereas for one cubic meter CNG, it was 11 kWh (DEFRA, 2012). The carbon emission costs of gasoline was \$0.06 per/liter and \$0.04 per/m<sup>3</sup> for CNG (CO<sub>2</sub> Calculator for Fossil Fuels). Other attributes including net weight, fuel tank capacity, capacity of vehicle, fixed and variable cost of operating a vehicle are provided in Table 3.



Table 3  
Characteristics of Vehicles

Vehicle	Net weight	Fuel tank capacity (CNG in $m^3$ , gasoline in Lit)	Freight capacity	Fixed Cost	Variable Cost (per Km)
$t_1$	5800	(30 60)	2000	120	0.05
$t_2$	3640	(30 60)	800	100	0.05
$t_3$	4820	(30 60)	2500	130	0.05

By using a CPLEX solver to solve the mathematical model, the optimal route required 472 km traveling. Using CNG solely, the truck consumed 41  $m^3$  whereas the capacity of the CNG tank is 30  $m^3$ . Hence, two strategies are available to respond to the mismatch:

**Strategy 1.** Proceed to a CNG station: In this case, the optimal traveling distance and CNG fuel consumption are 504 km and 43.8  $m^3$ , respectively. Accordingly, the additional traveling distance was 32 km and thus, it required additional CNG by 2.8  $m^3$ . The objective function value was \$291.1 (Figure 2).

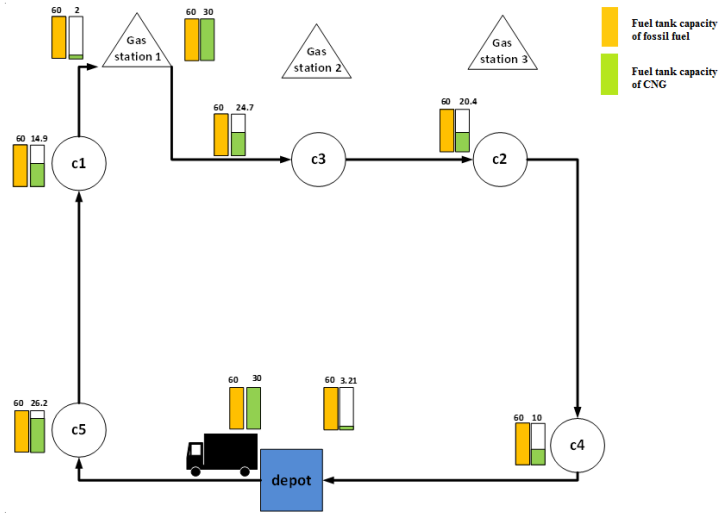


Fig. 1. The first resolution of the illustrative example (CNG station)

**Strategy 2.** Switch to gasoline: The route distance was 472 km and the truck consumed 29.4  $m^3$  CNG and 13.6

ltr of gasoline. The objective function value in this case was \$295.90 (Figure 3).

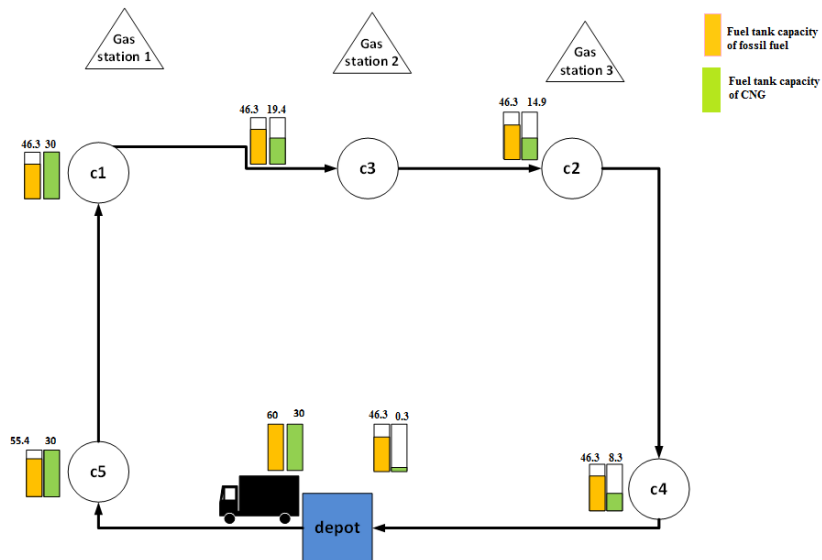


Fig. 3. The second resolution of the illustrative example (Using gasoline)

We further investigated the performance of the proposed model using a network of five customers and three bi-fuel vehicles. In following, the utilization of bi-fuel vehicles, we cross-examined with single-fuel vehicles, (i.e., gasoline). Additionally, we analyzed the sensitivity of each objective function component: different vehicles' average speed, the usage of each fuel type in percentage, and the number of visiting gas stations. The default

average speed was 40 km/hr for all vehicles. Finally, we assumed the average speeds of 60 and 70 km/hr in order to study the importance of average speed on a tuple of objectives. Other parameters in regard to the given network are summarized in Tables 3 and 4. Also, customer demand-related parameters are provided in Table 2.

Table 4  
Distances between nodes (Km)

depot	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$\omega_1$	$\omega_2$	$\omega_3$
	95	113	78	81	40	93	60	104
		123	77	169	125	22	88	150
			48	122	150	103	54	48
				118	117	59	23	74
					84	161	95	82
						127	99	134
							73	132
								63

The average service time of all customers and the refueling times at all stations were found to be 600s and 1200s, respectively. We solved the above working example by considering all components in totality. Subsequently, we considered each objective function, separately.

4.1. Full Objective function

The solution is summarized in Table 5, when all components in the objective function were considered. In this table, it is specified which arc in traveled by which vehicle. Moreover, the load of vehicle in each arc, used fuel, and trespassing from time restrictions are shown in Table 5. The optimal objective function value was

\$168.14, in which the vehicle uses only CNG fuel. Clearly, prolusion and fuel costs of CNG were lower vis-à-vis gasoline; thus, the solution was feasible. By contrast, truck  $t_1$  was used because its capacity was greater than the total demand. Although the fixed cost of truck  $t_1$  was lower than truck  $t_3$ , whose capacity also covered the total demand, its weight was heavier. Accordingly, the reason for using truck  $t_1$  in comparison to truck  $t_3$  was that the fixed cost of the truck usage was greater than the fuel and carbon emissions costs. The sequence of customers visited is shown in Table 5. As observed, two customers, namely nodes 3 and 1 were visited beyond their time windows.

Table 5  
Optimal solution considering all term of objective function

arc	truck #	load	traveled distance	used fuel	earliness (s)	lateness (s)
0-5	1	5800+1050	39.5	CNG (3.06)	0	0
5-4	1	5800+750	84.00	CNG (6.47)	0	0
4-2	1	5800+600	122.00	CNG (9.25)	0	0
2-3	1	5800+300	48.00	CNG (3.52)	0	4085
3-6	1	5800+150	58.50	CNG (4.21)	0	0
6-1	1	5800+150	21.50	CNG (1.54)	0	11240
1-0	1	5800+0	95.9	CNG (6.73)	0	0
Total			469.4	34.86	0	15325

Regarding the fuel consumption function, using arc 0-5 as an example, we show the calculation steps. The expected

consumed energy was 35857800.24 j (9.96 kw) while the produced energy of CNG, after multiplying the double

efficiency factor, was 33.70 kw. Thus, the amount of required CNG fuel was 3.06 m<sup>3</sup> (see Table 5). Because the vehicle visited a gas station for refueling, it caused a delay in the arrival time of customer 1. Hence, in this case, we observed that carbon emissions and fuel costs

were greater than the penalty cost associated with the late arrival.

Relying solely on gasoline as a single fuel, generating the optimal solutions shown in Table 6, where the objective function value was \$257.70; 12% greater than the case of bi-fuel. Data shown in this table can be interpreted similar to Table 5.

Table 6  
Optimal solution considering all terms of objective function (Gasoline fuel only)

arc	truck #	load	traveled distance	used fuel	earliness(s)	lateness(s)
0-5	1	5800+1050	39.5	Gasoline (3.76)	0	0
5-4	1	5800+750	84.00	Gasoline (7.73)	0	0
4-2	1	5800+600	122.00	Gasoline (11.06)	0	0
2-3	1	5800+300	48.00	Gasoline (4.20)	0	4084
3-1	1	5800+150	77.20	Gasoline (6.65)	0	9787
1-0	1	5800+0	95.10	Gasoline (8.05)	0	0
Total			465.8	41.45		13871

Although the objective function value was greater than the bi-fuel case, the vehicles did not need to travel to a gasoline station, resulting in added distance/time (465.8 km vs. 469.4 km). As evident in Table 6, a lesser marginal increase in distance/time reduced the propensity for later arrivals, in the case of customer 1.

#### 4.2. Transportation costs

Transportation cost comprises both fixed and variable vehicle costs, and fuel costs. In this case, the optimal objective function value was \$163.68 (see Table 7).

Although the route was identical with the optimal solution of having triple components in the objective function, transportation costs constituted the dominant component of the triple objective function (i.e. 97.3%, in which the ratio to fuel cost was 12.4%). However, if we utilized single fuel vehicles, the objective function value would be \$179.81 and the ratio of fuel cost was 20.29%, which was a greater ratio vis-à-vis bi-fuel vehicles. Consequently, the bi-fuel objective function value was 4.36% lower than using single fuel vehicles.

Table 7  
Optimal solution considering only transportation costs

arc	Truck #	load	Traveled distance	Used fuel	earliness(s)	lateness(s)
0-5	1	5800+1050	39.5	CNG (3.14)	0	0
5-4	1	5800+750	84.00	CNG (6.47)	0	0
4-2	1	5800+600	122.00	CNG (9.25)	0	0
2-3	1	5800+300	48.00	CNG (3.52)	0	4085
3-6	1	5800+150	58.50	CNG (4.21)	0	0
6-1	1	5800+150	21.50	CNG (1.54)	0	11240
1-0	1	5800+0	95.9	CNG (6.73)	0	0
Total			469.4	34.86		15325

#### 4.3. Service costs

In this case, the optimal objective required utilizing the available infrastructure (e.g. vehicles, fuels, etc.) in order to reduce service costs. The optimal objective function

value for both bi-fuel and single fuel vehicle cases was zero, in which only two vehicles were necessary for on-time delivery. The optimal solution is depicted in Table 8 (Similar to Tables 5-7).

Table 8  
Optimal solution considering service cost

arc	truck #	load	traveled distance	used fuel	earliness (s)	lateness (s)
0-2	1	5800+600	112.9	CNG (8.56)	0	0
2-3	1	5800+300	48.00	Gasoline (4.2)	0	0
3-1	1	5800+150	77.20	CNG (5.56)	0	0

1-0	1	5800+00	95.1	Gasoline (8.05)	0	0
0-4	2	3646+450	80.90	Gasoline (5.47)	0	0
4-5	2	3646+300	84	CNG (4.64)	0	0
5-0	2	3646+0	39.5	Gasoline (2.49)	0	0
Total			537.6	CNG: 18.76 gasoline: 20.21		

As shown in Table 8, the fuel type was not a distinguishing factor, since vehicles used both fuels with no priority. As bi-fuel vehicles can operate a longer range in comparison with single fuel type vehicles, reducing the need for frequent refueling, the results were reduced time wastage.

4.4. Carbon emission costs

If we consider carbon emission costs as a single objective, the optimal function value was \$1.149 and the optimal solution is shown in Table 9. In this case, the optimal solution comprises the utilization of two vehicles incurring a total CNG fuel of \$28.71. This was 20.51% less than the case of the triple components in the objective function. Conversely, if we used single fuel type vehicles, the optimal objective function value would be \$2.05 by utilizing two vehicles. In this instance, it is 43.95% greater than the case of bi-fuel.

Table 9  
Optimal solution considering Pollution cost

arc	truck #	load	traveled distance	used fuel	early time (s)	late time (s)
0-4	2	3646+750	80.90	CNG (4.78)	0	0
4-2	2	3646+600	122.00	CNG (7.05)	0	0
2-3	2	3646+300	48.00	CNG (2.65)	0	0
3-1	2	3646+150	77.00	CNG (4.17)	0	5357
1-0	2	3646+0	95.10	CNG (5.02)	0	0
0-5	3	4824+300	39.50	CNG (2.57)	0	0
5-0	3	4824+0	39.50	CNG (2.47)	0	0
Total			502	CNG: 28.71 Gasoline: 0		5357

4.5. Does speed matter?

Since speed is one of the key factors driving fuel efficiency, constituting the fuel consumption function, we studied the sensitivity of speed by solving the introduced

examples using 40, 60, and 70  $\frac{km}{hr}$  as the average speeds. The optimal solutions considering the average speeds of 60 and 70  $\frac{km}{hr}$  are shown in Tables 10 and 11, respectively. (See Table 5 for the average speed of 40  $\frac{km}{hr}$ ).

Table10  
Optimal solution with average speed of 60

arc	truck #	load	traveled distance	used fuel	early time (s)	late time (s)
0-5	1	5800+1050	39.5	CNG (3.63)	0	0
5-4	1	5800+750	84.00	CNG (7.52)	0	0
4-8	1	5800+600	82.00	CNG (7.26)	0	0
8-2	1	5800+600	48.00	CNG (4.24)	0	0
2-3	1	5800+300	48.00	CNG (4.11)	0	1290
3-1	1	5800+150	77.20	CNG (6.52)	0	0
1-0	1	5800+0	95.1	CNG (7.92)	0	0
Objective function value: 169.30			473.8	CNG: 41.20 Gasoline: 0		1290

Table 11  
Optimal solution with average speed of 70

arc	truck #	load	traveled distance	used fuel	early time (s)	late time (s)
0-5	1	5800+1050	39.5	CNG (4.95)	0	0
5-4	1	5800+750	84.00	CNG (10.32)	0	0
4-8	1	5800+600	82.00	CNG (10)	0	0
8-2	1	5800+600	48.00	CNG (5.84)	0	0

2-3	1	5800+300	48.00	Gasoline (6.83)	0	0
3-1	1	5800+150	77.20	CNG (9.10)	0	0
1-0	1	5800+0	95.1	CNG (11.1)	0	0
Objective function value:		473.8		CNG: 51.31		
181.97				Gasoline:6.83		

The difference between the objective function components at each level of the average speed is represented in Figure 4. As observed, the least optimal

objective function value was achieved when the average speed is  $40 \frac{km}{hr}$ .

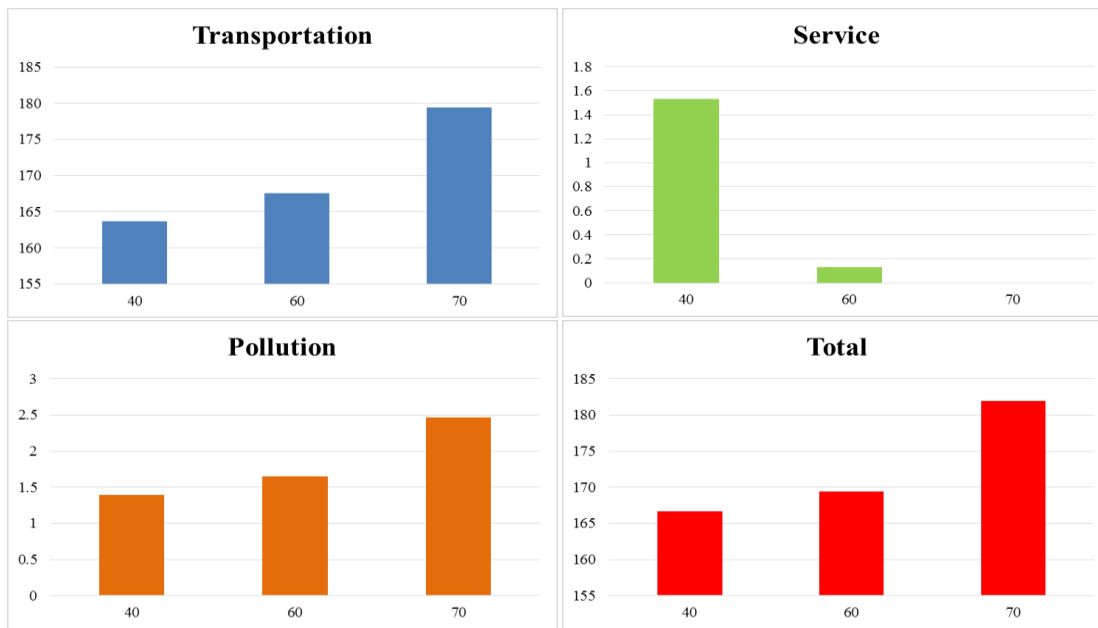


Fig. 4. The comparison between objective function values at different speed

#### 4.6. The importance of appropriate infrastructure

Generally, refueling imposes additional costs to the network, for example, through added traveled distance and time wastage. Here, we assumed such a condition for the network by considering highly congested gas stations,

requiring more time for refueling. We simulated this case by multiplying refueling times by five to dramatically increase the times for refueling. In this scenario, the optimal objective value was \$181.01 and the solution is presented in Table 12.

Table 12  
Optimal solution considering high congested gas station

arc	truck #	load	traveled distance	used fuel	earliness (s)	lateness (s)
0-5	1	5800+1050	39.5	CNG (3.14)	0	0
5-4	1	5800+750	84.00	CNG (6.47)	0	0
4-2	1	5800+600	122.00	CNG (9.25)	0	0
2-3	1	5800+300	48.00	CNG (3.52)	0	4084
3-1	1	5800+150	77.20	Gasoline (6.65)	0	9787
1-0	1	5800+0	95.10	CNG (6.73)	0	0
Total			465.8	CNG: 29.11 Gasoline: 6.65		13871

Although the objective function value is 7% greater than the solution in Table 5, it is 4% lower when compared with gasoline fuel, as in Table 6. Overall, providing an appropriate infrastructure (e.g. availability of gas stations) would be essential for a high performance network of AFVs or bi-fuel vehicles.

**5. Computational Analysis**

We followed the network structure approach described in Schneider et al. (2014) to carry out the computational analysis. In particular, we replaced the e-charging units with CNG/gasoline stations. Specifications of the pickup

trucks are summarized in Table 13 with the range parameters listed in Table 14. These parameters are adapted from Bektaş and Laporte (2011).

All instances were implemented using Microsoft visual C++ 2012 in conjunction with IBM ILOG CPLEX CONCERT Technology C++ API and by applying a default setting of CPLEX for optimality. The problems were solved using IBM ILOG CPLEX 12.6 32 bit edition on Windows 8.1, 64 bit environment with Intel core i7 4770K and 16 GB of RAM. Because the application had a memory limit of 3GB, we limited the computational time to 4000 seconds for each instance.

Table13  
Specifications of pickup trucks

Truck type	Fuel tank capacity (Gasoline)	Fuel tank (CNG)	Net weight (kg)	Max. load weight (kg)	Fixed acquisition cost
I	110	55	5800	2000	120
II	100	50	3646	800	100
III	120	60	4824	2500	130

In all instances, the assigned average speed was 40  $\frac{Km}{hr}$  and the basic information of each instance could be extracted from the associated string of codes. For example, (C15.G2.T3.F2) was a network of 15 customers (C) with two gas stations (G), three pickup trucks (T), and two fuel types (F). Computational results were collected in Table 15-17. In these tables obtained solutions with consideration of bi-fuel vehicles and Gasoline vehicles are reported, separately. What is more, each part of objective function and optimality gap are shown to make comparison between two scenarios more easily.

Table 14  
Range parameters

Parameter	Value
$\kappa^f$	\$0.04
	\$0.06
$p^f$	\$0.58
	\$0.88
$\pi^+$	\$0.0001
$\pi^-$	\$0.0002
$\psi^t$	\$0.05
$\zeta_i$	600, 1200
$\alpha_{ij}$	0.098
$\beta^t$	2.107
$\eta^f$	0.2x0.394
	0.2x0.412
$\varepsilon^f$	11x36x10 <sup>5</sup>
	8.8x36x10 <sup>5</sup>
$T_{max}$	k43200 (s)

Table 15  
Computational Results: R Series

Instance Name	Vehicle Type	Objectives					
		All	Gap	Transportation	Gap	Pollution	Gap
C5,G2,T3,F2	bi-fuel	139.144	0.00%	138.552	0.00%	0.483883	0.00%
	Gasoline	146.605	0.00%	145.543	0.00%	0.867638	0.00%
C5.G2.T3.F2	Gasoline	147.041	0.00%	145.961	0.00%	0.874905	0.00%
C5.G2.T3.F2	Gasoline	145.342	0.00%	144.33	0.00%	0.874759	0.00%
C5.G3.T3.F2	Gasoline	156.049	0.00%	154.6	0.00%	1.23229	0.00%
C10.G3.T3.F2	Gasoline	166.144	0.00%	164.722	0.00%	1.4226	0.00%
C10.G2.T3.F2	Gasoline	157.653	0.00%	156.573	0.00%	1.03391	0.00%
C10.G3.T3.F2	Gasoline	258.057	0.00%	256.564	0.00%	1.33118	0.00%
C10.G4.T3.F2	Gasoline	166.327	0.00%	161.671	0.00%	1.48774	0.00%
C15.G4.T3.F2	Gasoline	297.058	0.43%	295.58	0.00%	1.48348	14.77%
C15.G7.T3.F2	Gasoline	276.182	0.00%	274.441	0.00%	1.74094	3.71%

C15.G5.T3.F2	Gasoline	278.095	2.21%	281.389	4.71%	1.88866	20.68%
C15.G5.T3.F2	Gasoline	309.211	1.52%	305.056	0.00%	2.27093	8.55%
Avg.		5.23%		4.78%		44.35%	

Table 16  
Computational Results (Continued): C Series

Instance Name	Vehicle Type	Objectives					
		All	Gap	Transportation	Gap	Pollution	Gap
C5.G2.T3.F2	bi-fuel	144.362	0.00%	143.597	0	0.682806	0
	Gasoline	153.99	0.00%	152.619	0.00%	1.22433	0.00%
C5.G1.T3.F2	Gasoline	149.637	0.00%	148.447	0.00%	1.02158	0.00%
C5.G3.T3.F2	Gasoline	157.376	0.00%	155.883	0.00%	1.23646	0.00%
C5.G2.T3.F2	bi-fuel	142.18	0.00%	141.484	0.00%	0.620828	0.00%
	Gasoline	150.936	0.00%	149.689	0.00%	1.11319	0.00%
C10.G4.T3.F2	Gasoline	281.96	0.00%	278.723	0.00%	1.9412	0.00%
C10.G3.T3.F2	Gasoline	272.336	0.00%	270.292	0.00%	1.86079	0.00%
C10.G4.T3.F2	Gasoline	296.296	0.00%	294.458	0.00%	1.83803	0.00%
C10.G2.T3.F2	Gasoline	270.792	0.00%	268.789	0.00%	1.72141	0.00%
C15.G4.T3.F2	Gasoline	305.587	1.45%	303.369	0.00%	2.21751	12.02%
C15.G2.T3.F2	Gasoline	265.094	2.19%	269.359	5.22%	1.57625	9.19%
C15.G4.T3.F2	Gasoline	312.325	1.55%	309.86	0.00%	2.43237	11.45%
C15.G3.T3.F2	Gasoline	303.18	0.00%	301.082	0.00%	2.04649	10.37%
Avg.		5.23%		4.78%		44.35%	

All instances show greater objective function values for Gasoline vehicles. The difference in the carbon emission objective was significant, with an average of 44.23% increase. In addition, for all instances, the objective function values of service costs, as a single objective, were zero. A typical solution shows the adequacy of vehicle utilization, arriving at each customer within the predefined time window.

## 6. Conclusion

In this paper, we studied the class of Green vehicle routing problem (GVRP) to introduce the concept of utilizing bi-fuel (e.g. CNG and Gasoline) vehicles along the routes. Such vehicles are suitable for heterogeneous wide networks because of a more efficient usage of fuel tank capacity. Moreover, bi-fuel vehicles are more eco-friendly in comparison with fossil fuels. We proposed a mathematical model of bi-fuel VRP with linear equations to compute the fuel consumption of each fuel type. In order to analyze the performance of the proposed model, an in-depth illustrative example was provided and two strategies were explored to overcome the limitation of fuel capacity.

Based on the computational analysis, we found that employing bi-fuel vehicles contributed to a 44% reduction in carbon emission costs. The total distribution costs experienced a 6% reduction. Further, our study demonstrated an estimated 5% reduction in the overall transportation costs, derived through lower fuel costs as a consequence of using bi-fuel vehicles. Interestingly, we found paradoxical effects of average speed. On the one hand, higher speed reduced service costs. On the other hand, higher speed increased carbon emission costs. Therefore, firms need to consider the trade-offs when prioritizing objectives.

Given the increasing trend of adopting CNG and gasoline hybrids, due to the lower contribution to environmental pollution and lower fuel prices, our findings have important managerial and governmental implications. Notably, if service time is considered to be the dominant objective, then it is inevitable for firms to increase their size of vehicle fleet in order to fulfill the service time criteria, in terms of customers' demanded time windows. Thus, a critical issue for managers to focus attention on, relates to deciding the trade-off between carbon emissions and service costs. Given the bi-fuel vehicles' capability to operate more optimally over a longer range, through a

reduced frequency in refueling and time wastage, vis-à-vis single fuel type vehicles, such employment is especially suitable when a firm's customers prioritize time windows over other service dimensions.

From an institutional perspective, given the benefits associated with the use of bi-fuel vehicles, governments should consider equipping urban/suburban areas with the appropriate service infrastructure (e.g. gas stations) to promote adoption. Although bi-fuel vehicles could overcome the limitations of range due to a higher capacity



from their double fuel tanks, reducing the need for frequent refueling, governmental institutions should not simply rely on this benefit. The lack of service infrastructure induces inertia in the adoption of bi-fuel vehicles causing manufacturers to maintain bi-fuel vehicle prices at an uneconomical level for widespread implementation, without the corresponding adjustments in demand and supply functions, to exploit the benefits of these technological solutions. Because of the low ratio of bi-fuel vehicles to traditional gasoline-based vehicles, there is limited access to real-

world datasets to validate our model. However, we have enhanced the validity through sensitivity analysis and using a range of possible parameters to test the applicability of the model. Also, the fuel consumption function adopted in this study proved to only be an estimation of the actual fuel consumption on roads. As such, we have used a conservative measure for the fuel consumption and the reported results provided a lower estimate of the actual impact. Future study should be conducted around the possibility of replacing fuels.

Table 17  
Computational Results (Continued): RC Series

Instance Name	Vehicle Type	Objectives					
		All	Gap	Transportation	Gap	Pollution	Gap
C5,G3,T3,F2	bi-fuel	157.95	0	157.105	0	0.806271	0
	Gasoline	168.584	0	167.07	0	1.4457	0
C5,G3,T3,F2	bi-fuel	150.575	0	149.559	0	0.863366	0
	Gasoline	162.769	0	160.946	0	1.54809	0
C5,G3,T3,F2	bi-fuel	145.388	0	144.582	0	0.652255	0
	Gasoline	155.531	0	154.086	0	1.16954	0
C5,G2,T3,F2	bi-fuel	143.498	0	142.769	0	0.649127	0
	Gasoline	152.676	0	151.369	0	1.16394	0
C10,G3,T3,F2	bi-fuel	266.814	0	265.381	0	1.26942	0
	Gasoline	284.867	0	282.296	0	2.27619	0
C10,G3,T3,F2	bi-fuel	210.776	0.174121	169.08	0	1.16754	0
	Gasoline	227.506	0.159083	183.236	0	2.09349	0
C10,G3,T3,F2	bi-fuel	173.626	0.025172	162.152	0	0.999241	0
	Gasoline	187.08	0	174.012	0	1.7917	0
C10,G3,T3,F2	bi-fuel	172.484	0	168.633	0	1.18477	0
	Gasoline	231.437	0.186608	182.952	0	2.1244	0
C15,G4,T3,F2	bi-fuel	280.207	0.059233	273.217	0.026301	N/A	-
	Gasoline	305.906	0.102862	289.481	0.043416	2.32477	0.236757
C15,G4,T3,F2	Gasoline	328.924	0.100674	312.168	0.009442	2.59432	0.195465
C15,G4,T3,F2	Gasoline	312.771	0.039953	310.601	0.038982	2.48896	0.232561
C15,G6,T3,F2	Gasoline	312.153	0.048351	309.754	0.044655	2.91832	0.426919
Avg.		8.41%		6.23%		44.29%	
Total Avg.		6.34%		5.39%		44.23%	

While traversing an arc route. Given the potential contradictions in the objectives, an avenue for future research includes formulating the GVRP as a multi-objective mathematical model. It would also be interesting to extend this model for last-mile delivery operations within the emerging e-commerce and omnichannel context, experiencing tremendous growth in both the developed and developing countries (Lim, Rabinovich, Rogers, and Laseter, 2016). Notwithstanding,

the fuel consumption function could be extended to include additional options to capture vehicle and industry level intrinsic characteristics. For example, whether the existing understanding about fuel consumption function needs to be modified to cater for the advent of unmanned vehicles (e.g. drones and delivery robots) increasingly used for home delivery.

## References

- Association NGS. (2004). Natural Gas and the Environment Online: <http://www.naturalgas.org/environment/naturalgas.asp#greenhouse/> [Accessed on: 12/03/08].
- Azadeh, A., Ghaderi, S. F., Pashapour, S., Keramati, A., Malek, M. R., & Esmizadeh, M. (2017). A unique fuzzy multivariate modeling approach for performance optimization of maintenance workshops with cognitive factors. *The International Journal of Advanced Manufacturing Technology*, 90(1-4), 499-525.
- Azadeh, A., & Farrokhi-Asl, H. (2019). The close–open mixed multi depot vehicle routing problem considering internal and external fleet of vehicles. *Transportation Letters*, 11(2), 78-92.
- Barth, M., and Boriboonsomsin, K. (2009). Energy and emissions impacts of a freeway-based dynamic eco-driving system. *Transportation Research Part D: Transport and Environment*, 14(6), 400-410.
- Barth, M., Younglove, T., and Scora, G. (2005). Development of a heavy-duty diesel modal emissions and fuel consumption model. *California Partners for Advanced Transit and Highways (PATH)*.
- Bektaş, T., and Laporte, G. (2011). The pollution-routing problem. *Transportation Research Part B: Methodological*, 45(8), 1232-1250.
- Bynum, C., Sze, C., Kearns, D., Polovick, B., & Simon, K. (2018). An examination of a voluntary policy model to effect behavioral change and influence interactions and decision making in the freight sector. *Transportation Research Part D: Transport and Environment*, 61, 19-32.
- Christopher, M. (2005). *Logistics and supply chain management: creating value-adding networks*. Pearson education.
- CO2 Calculator for Fossil Fuels. <http://www.environment-watch.co.uk/co2.cgi>.
- Daneshzand, F. (2011). The vehicle-routing problem. *Logistics Operations and Management*, 8, 127-153.
- Dantzig, G. B., and Ramser, J. H. (1959). The truck dispatching problem. *Management science*, 6(1), 80-91.
- De Grancy, G. S., and Reimann, M. (2015). Evaluating two new heuristics for constructing customer clusters in a VRPTW with multiple service workers. *Central European Journal of Operations Research*, 23(2), 479-500.
- DEFRA (2012) 2012 Guidelines to Defra / DECC's GHG Conversion Factors for Company Reporting. [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/69554/pb13773-ghg-conversion-factors-2012.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/69554/pb13773-ghg-conversion-factors-2012.pdf)
- Dell'Amico, M., Monaci, M., Pagani, C., and Vigo, D. (2007). Heuristic approaches for the fleet size and mix vehicle routing problem with time windows. *Transportation Science*, 41(4), 516-526.
- Demir, E., Bektaş, T., and Laporte, G. (2012). An adaptive large neighborhood search heuristic for the pollution-routing problem. *European Journal of Operational Research*, 223(2), 346-359.
- Demir, E., Bektaş, T., and Laporte, G. (2014). The bi-objective pollution-routing problem. *European Journal of Operational Research*, 232(3), 464-478.
- Erdoğan, S., and Miller-Hooks, E. (2012). A green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, 48(1), 100-114.
- Farrokhi-Asl, H., Makui, A., Jabbarzadeh, A., & Barzinpour, F. (2018). Solving a multi-objective sustainable waste collection problem considering a new collection network. *Operational Research*, 1-39.
- Fuchs, E. F., and Masoum, M. A. (2011). *Power conversion of renewable energy systems*. Springer Science and Business Media.
- Golden B., Assad A., Levy L., Gheysens F. (1984). The fleet size and mix vehicle routing problem. *Computers and Operations Research*, 11:49-66
- Guerriero, F., Surace, R., Loscri, V., and Natalizio, E. (2014). A multi-objective approach for unmanned aerial vehicle routing problem with soft time windows constraints. *Applied Mathematical Modelling*, 38(3), 839-852.
- Huang, X., Wang, Y., Xing, Z., and Du, K. (2016). Emission factors of air pollutants from CNG-gasoline bi-fuel vehicles: Part II. CO, HC and NO x. *Science of the Total Environment*, 565, 698-705.
- Jabali, O., Woensel, T., and de Kok, A. G. (2012). Analysis of travel times and CO2 emissions in time-dependent vehicle routing. *Production and Operations Management*, 21(6), 1060-1074.
- Kara, I., Kara, B. Y., and Yetis, M. K. (2007, August). Energy minimizing vehicle routing problem. In *International Conference on Combinatorial Optimization and Applications* (pp. 62-71). Springer Berlin Heidelberg.
- Knörr, W. (2011). *EcoTransIT: Ecological transport information tool for worldwide transports - Methodology and data update*. Heidelberg, Germany: Institut für Energie (ifeu) und Umweltforschung Heidelberg GmbH.
- Koç, Ç., and Karaoglan, I. (2016). The green vehicle routing problem: A heuristic based exact solution approach. *Applied Soft Computing*, 39, 154-164.
- Lin, C., Choy, K. L., Ho, G. T., Chung, S. H., and Lam, H. Y. (2014). Survey of green vehicle routing problem: past and future trends. *Expert Systems with Applications*, 41(4), 1118-1138.
- Lim, S.F.W.T., Rabinovich, E., Rogers, D.S. and Laseter, T.M. (2016). Last-mile supply network distribution in omni-channel retailing: A configuration-based typology. *Foundations and Trends in Technology, Information and Operations Management*, 10(1), 1-87.
- Lowe, I. (2016). The electric vehicle challenge. *Australasian Science*, 37(4), 49.

- Maden, W., Eglese, R., and Black, D. (2010). Vehicle routing and scheduling with time-varying data: A case study. *Journal of the Operational Research Society*, 61(3), 515-522.
- Min, H. (1991). A multiobjective vehicle routing problem with soft time windows: the case of a public library distribution system. *Socio-Economic Planning Sciences*, 25(3), 179-188.
- Omidvar, A., and Tavakkoli-Moghaddam, R. (2012, September). Sustainable vehicle routing: Strategies for congestion management and refueling scheduling. In *Energy Conference and Exhibition (ENERGYCON), 2012 IEEE International* (pp. 1089-1094). IEEE.
- Palmer, A. (2007). The development of an integrated routing and carbon dioxide emissions model for goods vehicles. Bedford: Cranfield University, Available from: <http://hdl.handle.net/1826/2547>.
- Pradenas, L., Oportus, B., and Parada, V. (2013). Mitigation of greenhouse gas emissions in vehicle routing problems with backhauling. *Expert Systems with Applications*, 40(8), 2985-2991.
- Qureshi, A. G., Taniguchi, E., and Yamada, T. (2009). An exact solution approach for vehicle routing and scheduling problems with soft time windows. *Transportation Research Part E: Logistics and Transportation Review*, 45(6), 960-977.
- Quak, H., and Dekoster, M. (2007). Exploring retailers' sensitivity to local sustainability policies. *Journal of Operations Management*, 25, 1103-1122.
- Rabbani, M., Ramezankhani, M. J., Farrokhi-Asl, H., and Farshbaf-Geranmayeh, A. (2015). Vehicle Routing with Time Windows and Customer Selection for Perishable Goods. *International Journal of Supply and Operations Management*, 2(2), 700-719.
- Rabbani, M., Farrokhi-asl, H., and Rafiei, H. (2016). A hybrid genetic algorithm for waste collection problem by heterogeneous fleet of vehicles with multiple separated compartments. *Journal of Intelligent and Fuzzy Systems*, 30(3), 1817-1830.
- Rabbani, M., Heidari, R., Farrokhi-Asl, H., & Rahimi, N. (2018). Using metaheuristic algorithms to solve a multi-objective industrial hazardous waste location-routing problem considering incompatible waste types. *Journal of Cleaner Production*, 170, 227-241.
- Russell, R. A. (1977). Technical Note—An Effective Heuristic for the M-Tour Traveling Salesman Problem with Some Side Conditions. *Operations Research*, 25(3), 517-524.
- Salimifard, K., and Raeesi, R. (2014). A green routing problem: optimising CO2 emissions and costs from a bi-fuel vehicle fleet. *International Journal of Advanced Operations Management*, 6(1), 27-57.
- Salhi, S., Sari, M., Saidi, D., and Touati, N. A. C. (1992). Adaptation of some vehicle fleet mix heuristics. *Omega*, 20(5-6), 653-660.
- Schneider, M., Stenger, A., and Goeke, D. (2014). The electric vehicle-routing problem with time windows and recharging stations. *Transportation Science*, 48(4), 500-520.
- Sexton, T. R., and Choi, Y. M. (1986). Pickup and delivery of partial loads with "soft" time windows. *American Journal of Mathematical and Management Sciences*, 6(3-4), 369-398.
- Shahraeeni, M., Ahmed, S., Malek, K., Van Drimmelen, B., and Kjeang, E. (2015). Life cycle emissions and cost of transportation systems: Case study on diesel and natural gas for light duty trucks in municipal fleet operations. *Journal of Natural Gas Science and Engineering*, 24, 26-34.
- Solomon, M. M. (1987). Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations research*, 35(2), 254-265.
- Taillard, É. D. (1999). A heuristic column generation method for the heterogeneous fleet VRP. *RAIRO-Operations Research*, 33(1), 1-14.
- Toro, E., Franco, J., Echeverri, M., Guimarães, F., and Rendón, R. (2017). Green open location-routing problem considering economic and environmental costs. *International Journal of Industrial Engineering Computations*, 8(2), 203-216.
- Wilson, R. M., and Gilligan, C. (2012). *Strategic marketing management*. Routledge.
- Xiao, Y., Zhao, Q., Kaku, I., and Xu, Y. (2012). Development of a fuel consumption optimization model for the capacitated vehicle routing problem. *Computers and Operations Research*, 39(7), 1419-1431.
- Yang, C., McCollum, D., McCarthy, R., and Leighty, W. (2009). Meeting an 80% reduction in greenhouse gas emissions from transportation by 2050: A case study in California. *Transportation Research Part D: Transport and Environment*, 14(3), 147-156.
- Yavuz, M., Oztaysi, B., Onar, S. C., and Kahraman, C. (2015). Multi-criteria evaluation of alternative-fuel vehicles via a hierarchical hesitant fuzzy linguistic model. *Expert Systems with Applications*, 42(5), 2835-2848.
- Zhang, L. H., and Wang, M. Y. (2013). Study on a Multi-Depot and Heterogeneous-Vehicle Open Vehicle Routing Problem to Reduce Fuel Consumption. In *Applied Mechanics and Materials* (Vol. 336, pp. 2567-2571). Trans Tech Publication.

**This article can be cited:** Manavizadeha, N., Farrokhi-Asl H. & W.T. Lim, S.F. (2021). A New Mathematical Model for the Green Vehicle Routing Problem by Considering a Bi-Fuel Mixed Vehicle Fleet. *Journal of Optimization in Industrial Engineering*. 13 (2), 165-183.

[http://www.qjie.ir/article\\_671447.html](http://www.qjie.ir/article_671447.html)  
DOI:10.22094/JOIE.2020.1871922.1667

