Modeling and Optimization of a Tri-objective Transportation-Location-Routing Problem considering route reliability: using MOGWO, MOPSO, MOWCA, and NSGA-II

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Abstract

In this research, a tri-objective mathematical model is proposed for the Transportation-Location-Routing problem. The model considers a three-echelon supply chain and aims to minimize total costs, maximize the minimum reliability of the traveled routes and establish a well-balanced set of routes. In order to solve the proposed model, four metaheuristic algorithms, including Multi-Objective Grey Wolf Optimizer (MOGWO), Multi-Objective Water Cycle Algorithm (MOWCA), Multi-objective Particle Swarm Optimization (MOPSO) and Non-Dominated Sorting Genetic Algorithm- II (NSGA-II) are developed. The performance of the algorithms is evaluated by solving various test problems in small, medium, and large scale. Four performance measures, including Diversity, Hypervolume, Number of Non-dominated Solutions, and CPU-Time, are considered to evaluate the effectiveness of the algorithms. In the end, the superior algorithm is determined by Technique for Order of Preference by Similarity to Ideal Solution method.

Keywords: Transportation-Location-Routing; Reliability; Multi-Objective Grey Wolf Optimizer; Multi-Objective Water Cycle Algorithm; Multi-objective Particle Swarm Optimization; Non-Dominated Sorting Genetic Algorithm- II

1. Introduction

The distribution network in the supply chain management consists of operations related to the transportation of final products and distrusting them to the final clients. Decisions about the location of distribution centers and distribution of products among clients are the two main challenges that should be addressed in the distribution systems (Martínez-Salazar et al., 2014). To address decisions about location and routing in the supply chain management, two classical models, including Facility Location Problem (FLP) and Vehicle Routing Problem (VRP) should be combined, which result in Location Routing Problem (LRP). These models are widely applied to solve real-world problems in the blood supply chain, food supply chain, humanitarian relief, etc. Both FLP and VRP are known as NP-Hard problems. Therefore, the resulting LRP formulation is even more complex (Cornejo et al., 1977; Karp et al., 1972; Tuzun and Burke, 1999).

As shown in the literature, solving FLP and VRP models separately results in sub-optimal solutions. The decisions related to the location of facilities are at the strategic level. However, those related to the routing are at the tactical level. Thus, a comprehensive model that considers both decisions is of great importance. In addition to considering the complexity of the resulting model, it is needed to As mentioned earlier, the LRP model is NP-Hard. Therefore, developing an efficient solution algorithm for this problem is a challenging task. Many researchers have

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LRP problem. One of the most commonly used solution approaches to solve the NP-Hard LRP model is metaheuristics. Metaheuristics have several advantages over traditional techniques. The first and most important advantage of the metaheuristic algorithms over the traditional optimization methods is that they do not need gradient information to solve optimization problems. Second, using exploration ability, metaheuristics can decrease the probability of trapping in local optima (Khalilpourazari et al., 2020).

In recent years, many researchers have aimed to implement metaheuristic algorithms in order to solve LRP models such as Tabu Search (TS) (Albareda-Sambola et al., 2001; Albareda-Sambola et al., 2005, Caballero et al., 2007; Lin and Kwok, 2006; Tuzun and Burke, 1999; Wang et al., 2005), Simulated Annealing (SA) (Wu et al., 2002), Particle Swarm Optimization (PSO) (Peng and Bai, 2006), multiple ant colony optimization algorithm (MACO) (Ting and Chen, 2013), variable neighborhood search (VNS) (Jrbou et al., 2013), and some hybrid metaheuristics, such as a hybrid PSO with Path Relinking (Marinakis and Marinaki, propose efficient solution techniques to solve complex LRP.2008), an algorithm combining Simulated Annealing with an Ant Colony System (Khalilpourazari et al., 2020), a GRASP algorithm complemented by Path Relinking (Prins et al., 2006).

In recent years, Ghatreh Samani and Hosseini-Motlagh (2017) studied the LRP with simultaneous pickups and deliveries. In order to increase the applicability of the proposed model, the authors considered uncertainty in the formulation. The researchers proposed a MIP for the
problem and solved it using GA and SA. Adrang et al. (2020) have raised the problem of location routing for emergency medical services in the event of a disaster. This problem is formulated as a two-objective mixed-integer linear programming model (MILP). Brandão (2020) investigated the multi-depot open routing problem. The author developed a MIP for the problem and solved it using an iterated local search algorithm. Manavizadeh et al. (2020) In their paper, they present a mathematical model for the Green Vehicle Routing (GVRP) problem, combining dual-fuel pickup trucks (natural gas and gasoline) in a fleet of combined vehicles. Hosseini-Motlagh et al. (2020) proposed a robust model for the inventory routing problem. The authors also considered several depots in their model. The authors used a robust possibilistic approach to formulate the uncertainties in the model.

The main drawback and limitation in the previous studies was considering a single-stage LRP problem. Thus, in recent years researchers have aimed to develop more realistic models by considering multi-echelon supply chains such as: (Samani et al., 2019; Ambrosino and Scutella, 2005; Boccia et al., 2010; Contardo et al., 2012; Lashine et al., 2006; Hidayatul et al., 2019; Haeri et al., 2020; Hosseini-Motlagh et al., 2020; Haeri et al., 2020). One of the main challenges in the previous studies is that the authors claimed that the solution of the proposed models takes several hours for small scale problems and even days for large scale problems. Thus, proposing an efficient solution method for the LRP problem is still an issue.

In supply chain management and distribution systems, most decision-makers seek to optimize several objectives simultaneously, for instance, minimization of costs, minimization of total transportation time, or maximizing the reliability of the network. Therefore, developing new models that can consider several objectives is essential. In this regard, Martínez-Salazar et al. Proposed a new bi-objective mathematical model for the LRP problem considering transportation decisions, which is called Transportation Location Routing Problem (TLRP). The authors considered two objectives that aimed to minimize total network costs as well as establish well-balanced routes. They proposed two metaheuristics to solve the problem and showed that their proposed algorithms perform efficiently in large scale. In this paper, the proposed model by Martínez-Salazar et al. is modified by adding a new objective function that aims to maximize the minimum reliability of the routes in the routing stage (Martínez-Salazar et al., 2014). Reliability is one of the most critical objective functions in distribution systems in the majority of real-world applications such as disaster relief distribution, blood supply chain network design, disaster management, etc. Therefore, considering this critical objective is essential in TLRP.

Since the proposed model in this research is a tri-objective mathematical model, developing efficient solution methods to solve the problem is of great importance. In this research, the objective functions are in conflict. Thus, only efficient (Pareto) solutions are acceptable. These efficient solutions help the decision-makers to choose their preferred solutions and are of great importance. In this regard, in order to solve the proposed tri-objective model and obtain efficient Pareto optimal solutions, an efficient solution methodology called Multi-Objective Grey Wolf Optimizer is utilized, and its efficiency is evaluated and compared with well-known efficient algorithms such as Multi-Objective Water Cycle Algorithm (MOWCA), Multi-objective Particle Swarm Optimization (MOPSO) and Non-Dominated Sorting Genetic Algorithm-II (NSGA-II). Grey Wolf Optimizer (GWO) is a novel and recently developed metaheuristic algorithm which has been implemented on various research disciplines (Chaman-Motlagh, 2015; Emary et al., 2014; Kamboj et al., 2016; Khalilpourazari et al., 2019; Mohammadi and Khalilpourazari, 2017; Khalilpourazari and Mohammadi, 2016; Komaki and Kayvanfar, 2015; Khalilpourazari et al., 2019; Precup et al., 2017; Sharma and Saikia, 2015; Song et al., 2015; Sulaiman et al., 2015). To the best of our knowledge, this is the first time that the GWO is implemented and adopted to solve a complex TLRP. The performance of the algorithms is evaluated by solving various test problems within different test instances in small, medium, and large scale. Four performance measures, including Diversity, Hypervolume, Number of Non-dominated Solutions, and CPU-Time, are considered to evaluate the effectiveness of the algorithms. In the end, the superior algorithm is determined by Technique for Order of Preference by Similarity to Ideal Solution method.

2. Problem Definition

The TLRP considers a three-echelon supply chain, which consists of suppliers, City Distribution Centers, and clients (demand points). The final products are produced in plants and transported to CDCs using different transportation means with variant capacities (Martínez-Salazar et al., 2014). Then, products in the CDCs are used to satisfy the clients’ demand. The problem has some assumptions which are listed below.

- The production capacity of plants is limited.
- The capacity of transportation means that carry products from plants to CDCs is limited.
- The establishment costs of a CDC at each potential location is different.
- The capacity of vehicles that transport products from CDCs to clients is limited.
- Each client should be allocated to precisely one CDC.
- The demand of each client is known.

In the following, a tri-objective mathematical model is presented for the problem. The model aims to minimize total supply chain costs, the establishment of a well-balanced set of routes, and maximizing the minimum reliability of routes. By solving the proposed three objective mathematical model, the following decisions can be made.

- Optimal location of CDCs.
- The optimal number of required CDCs.
- The number of transportation means required to transport products from each plant to each CDC.
- The number of products transported from each plant to each CDC.
- Allocation of clients to CDCs.
- Optimal transportation routes.

The following sets, notations, and decision variables are considered in order to present the mathematical model of the problem.

### Sets

- \( k = 1, 2, \ldots, p \): Set of production plants
- \( j = 1, 2, \ldots, m \): Set of CDCs
- \( i = 1, 2, \ldots, n \): Set of clients
- \( l = i \): Set of clients
- \( q = i \cup j \): Set of CDCs and clients
- \( h = i \cup j \): Set of CDCs and clients

### Parameters

- \( a_k \): Production capacity of plant \( k \)
- \( d_{kj} \): Cost of sending one truck from plant \( k \) to CDC \( j \)
- \( R \): Truck capacity used for transportation between plants and CDCs
- \( g_j \): Fixed establishment cost of a CDC at potential location \( j \)
- \( b_j \): Capacity of CDC located at location \( j \)
- \( h_i \): Demand of client \( i \)
- \( c_{ij} \): Cost of visiting client or CDC \( j \) right after client or CDC \( i \) in the route stage
- \( Q \): Vehicle capacity in routing stage
- \( \tau_{ij} \): Traveling distance from client or CDC \( i \) to client or CDC \( j \)
- \( T \): Maximum allowable traveling distance vehicles in routing stage
- \( r_{hq} \): Reliability of route between facility \( h \) and facility \( q \)

### Decision Variables

- \( v_{kj} \): Amount of product transported from plant \( k \) to CDC \( j \)
- \( w_{kj} \): Number of trucks sent from plant \( k \) to CDC \( j \)
- \( y_j \): \( 1 \) if CDC \( j \) is opened; otherwise zero
- \( z_{ij} \): \( 1 \) if client \( i \) is allocated to CDC \( j \); otherwise zero
- \( x_{0ij} \): \( 1 \) if client \( i \) is the first client on any route starting from CDC \( j \)
- \( x_{lj} \): \( 1 \) if client \( i \) is the last client on any route starting from CDC \( j \)
- \( x_{ilj} \): \( 1 \) if client \( i \) is visited right after client \( l \) on any route of CDC \( j \); otherwise zero

### Decision Variables

\( u_i \): Vehicles capacity and sub-tour elimination variables.
\( e_i \): Maximum distance constraint variables.
\( L_{\text{min}} \): Shortest route length
\( L_{\text{max}} \): Longest route length
\( \text{reliability}_{hq} \): \( r_{hq} \) if \( X_{hq} \) is equal to 1; otherwise 1

The first objective function aims to minimize total supply chain costs, including fixed establishment costs of CDCs and transportation costs in the first echelon and routing costs in the second echelon of the supply chain. The total cost function of the considered supply chain can be presented as follows.

\[
\min z_1 = \sum_{k} \sum_{l} d_{lk} w_{k} + \sum_{i} b_{i} y_{i} + \sum_{j} \left( g_{j} + \sum_{l} \left( x_{0lj} + x_{lj} \right) \right) + \sum_{i} \sum_{j} c_{ij} x_{ij} \tag{1}
\]

The second objective function aims to establish well-balanced routes. This objective function is presented as follows [28]:

\[
\min z_2 = L_{\text{max}} - L_{\text{min}} \tag{2}
\]

In which \( L_{\text{min}} \) is the length of the shortest route and \( L_{\text{max}} \) is the length of the longest route.

The third objective function targets maximization of the minimum reliability of the established routes. This objective function is formulated as follows.

\[
\max z_3 = \min \left\{ \prod_{h} \prod_{q} \text{reliability}_{hq} \right\} \tag{3}
\]

In order to calculate the value of the variable \( \text{reliability}_{hq} \), the following constraints are considered.

\[
\text{reliability}_{hq} \geq (1 - x_{hq}) \tag{4}
\]

\[
\text{reliability}_{hq} \leq r_{hq} x_{hq} + (1 - x_{hq}) \tag{5}
\]

\[
\text{reliability}_{hq} \geq r_{hq} x_{hq} \tag{6}
\]

The above-mentioned constraints assign a value equal to \( r_{hq} \) selected routes and 1 to available routes. The production capacity of plant \( k \) is presented as follows.

\[
\sum_{j} v_{kj} \leq a_k \tag{7}
\]

The following constraint calculates the number of requires trucks to transport products from each plant to each CDC.

\[
w_{kj} \geq \frac{v_{kj}}{R} \tag{8}
\]

Where R is the capacity of trucks in the routing stage and \( w_{kj} \) is a positive integer variable, it is necessary to ensure that the total amount of transported products to a specific CDC does not exceed its capacity. For this purpose, the following constraint is considered.

\[
\sum_{k} v_{kj} \leq b_{j} y_{j} \tag{9}
\]
The total amount of transported products to each CDC is equal to the total demand of clients assigned to that CDC. The following constraint is considered to ensure this.
\[ \sum_{h} v_{ih} = \sum_{j} z_{ij} h_{i} \]  
(10)

The following constraint guarantees that each client is assigned to precisely one CDC.
\[ \sum_{j} z_{ij} = 1 \]  
(11)

The following constraint ensures that if a vehicle leaves a CDC, it should return to the same CDC.
\[ \sum_{h} x_{hij} = z_{ij} \]  
(12)

\[ \sum_{i} x_{ij} = 1 \]  
(13)

In the considered supply chain, it should be ensured that each client is visited immediately after exactly one CDC or after another client. On the other hand, some constraints should be considered to construct routes only between clients assigned to the same CDC.
\[ \sum_{h} x_{hij} = z_{ij} \]  
(14)

\[ \sum_{h} x_{hij} = z_{ij} \]  
(15)

In order to prevent exceeding vehicle capacities and avoid the generation of sub-tours, the following constraints are considered.
\[ u_{i} - u_{i} + Q \sum_{j} x_{ij} \leq Q - h_{i} \]  
(16)

\[ h_{i} \leq u_{i} \leq Q \]  
(17)

In order to calculate the value of the second objective function, the following constraints are considered [28].
\[ e_{i} - e_{i} + (T + \tau_{ij}) \sum_{j} x_{ij} + (T + \tau_{ij}) \sum_{j} x_{ij} \leq T \]  
(18)

\[ \sum_{j} x_{ij} \tau_{ij} \leq e_{i} \leq T + \sum_{j} x_{ij} (\tau_{ij} - T) \]  
(19)

\[ e_{i} \leq T - \sum_{j} \tau_{ij} x_{ij} \]  
(20)

\[ L_{\text{max}} \geq e_{i} + \sum_{j} \tau_{ij} x_{ij} \]  
(21)

\[ L_{\text{min}} \leq e_{i} + \sum_{j} (\tau_{ij} - T) x_{ij} + T \]  
(22)

The following constraints show the decision variables and their possible values.
\[ e_{i}, u_{i}, v_{ij} \geq 0 \]  
(23)

\[ y_{ij}, z_{ij}, x_{hij} \in \{0,1\} \]  
(24)

\[ w_{ij} \geq 0, \text{ int} \]  
(25)

\[ 0 < \text{reliability}_{\text{req}} \leq 1 \]  
(26)

3. Solution Methodology

The TLRP model presented in this paper consists of decisions related to the location of CDCs and routing. Both location and routing decisions are known as Np-hard problems in the literature. Therefore, the proposed model in this research is Np-hard. Besides, the proposed model in this research is a tri-objective model. For a multi-objective model, the ideal solution is a solution that minimizes (maximizes) the minimization (maximization) objective functions simultaneously. Since, in this research, the objective functions are in conflict, only efficient (Pareto) solutions are acceptable (Fazli-Khalaf et al., 2017; Pasandideh and Khalilpourazari, 2018; Khalilpourazari et al., 2019). These efficient solutions help the decision-makers to choose their preferred solutions and are of great importance. Therefore, in order to solve the proposed tri-objective model and obtain efficient Pareto optimal solutions, an efficient solution methodology called Multi-Objective Grey Wolf Optimizer is utilized. Its efficiency is evaluated and compared with well-known efficient algorithms such as Multi-Objective Particle Swarm Optimization (MOPSO), Multi-objective Water Cycle Algorithm (MOWCA), Multi-objective Non-Dominated Sorting Genetic Algorithm II (NSGA-II).

Grey Wolf Optimizer (GWO) is a novel and recently developed metaheuristic algorithm which has been implemented on various research disciplines (Chaman-Motlagh, 2015; Emary et al., 2014; Kamboj et al., 2017; Khalilpourazari et al., 2019). Grey Wolf Optimizer (GWO) mimics the behavior of Grey Wolves while hunting in order to optimize complex problems. Grey wolves usually live in a pack consisting of 5-12 grey wolves (Khalilpourazari and Pasandideh, 2019). One of the most interesting things about grey wolves is their social hierarchy, which is showed graphically in Figure 1.

3.1. Grey wolf optimizer

Grey Wolf Optimizer (GWO) was first proposed by Mirjalili et al., (2014). The GWO mimics the behavior of the Grey Wolves while hunting in order to optimize complex problems. Grey wolves usually live in a pack consisting of 5-12 grey wolves (Khalilpourazari and Pasandideh, 2019). One of the most interesting things about grey wolves is their social hierarchy, which is showed graphically in Figure 1.
Alpha, Beta, and Delta wolves are the most dominant wolves in the pack, respectively. The alpha wolf acts as the leader and is responsible for making decisions. The beta wolf is the wolf who helps the alpha in managing the pack. The omega wolves are lowest in rank and must follow Alpha, Beta, and Delta wolves. The other wolves are called Delta, who help the Alpha and Beta in managing the pack. In the GWO, the positions of dominant wolves are considered as the best approximation of the position of prey (optimal point). So, the GWO updates the position of other wolves regarding the positions of Alpha, Beta, and Delta. This process is called hunting, which is illustrated in the following.

In nature, the grey wolves start the hunting process by encircling the prey. In GWO, the hunting process also starts with encircling the prey. To encircle the prey, the following formulation is used to update the position of the grey wolves (Khalilpourazari and Pasandideh, 2019).

$$\vec{D} = [\vec{C} \cdot \vec{X}_{\rho} (t) - \vec{X}(t)]$$  

(27)

$$\vec{X}(t + 1) = \vec{X}_{\rho} (t) - \vec{A} \cdot \vec{D}$$  

(28)

Where $\vec{C}$ and $\vec{A}$ are coefficients, $\vec{X}_{\rho}$ is the position of prey and $\vec{X}$ is the position of the grey wolves (Mirjalili et al., 2014). In the above formulations, Parameter $\vec{A}$ has a prominent role since it determines the search radius of the grey wolves. These coefficients are calculated in each iteration of the GWO as follows.

$$\vec{A} = 2 \vec{\alpha} \cdot \vec{r}_1 - \vec{\alpha}$$  

(29)

$$\vec{C} = 2 \cdot \vec{r}_2$$  

(30)

Where $\vec{\alpha}$ decreases from two to zero throughout iterations. $\vec{r}_1, \vec{r}_2$ parameters are randomly generated numbers between 0 and 1. The above formulas allow the grey wolves to update their positions around the prey, efficiently.

As mentioned earlier, the omega wolves update their position based on the position of the dominant wolves. The following equations are considered to update the position of the omega wolves in GWO.

$$\vec{D}_\sigma = [\vec{C}_1 \cdot \vec{X}_\sigma - \vec{X}]$$  

$$\vec{D}_\rho = [\vec{C}_2 \cdot \vec{X}_\rho - \vec{X}]$$  

$$\vec{D}_\delta = [\vec{C}_1 \cdot \vec{X}_\delta - \vec{X}]$$  

(31)

$$\vec{X}_\sigma = \vec{X}_\sigma - A_\sigma \vec{D}_\sigma$$  

$$\vec{X}_\rho = \vec{X}_\rho - A_\rho \vec{D}_\rho$$  

$$\vec{X}_\delta = \vec{X}_\delta - A_\delta \vec{D}_\delta$$  

(32)

$$\vec{X}(t + 1) = \frac{\vec{X}_1 + \vec{X}_\rho + \vec{X}_\delta}{3}$$  

(33)

By decreasing the value of the $\vec{\alpha}$ (search radius) over the course of iterations, GWO makes a proper trade-off between exploration and exploitation during the optimization process (Khalilpourazari and Mohammadi, 2018).

In order to solve multi-objective problems using GWO, some modifications are needed. So, first, an external archive is added to basic GWO to maintain the obtained non-dominated Pareto solutions. By adding the archive to GWO, three cases may occur during optimization (Mirjalili et al., 2016)

1. A recently found Pareto solution can be dominated by at least one of the current solutions in the archive. In this case, the recently found solution will be ignored.

2. The new solution dominates one or more solutions in the archive. In this case, the dominated solutions are eliminated from the archive, and the new solution will be added to the archive.

3. If neither the new solution nor archive members dominate each other, the new solution should be added to the archive.

4. If the archive is full, the grid mechanism should be run to re-arrange the segmentation of the objective space to find the most crowded segment, the best segment to eliminate a solution from.

Second, grid mechanism and leader selection techniques are added to basic GWO in order to determine dominant wolves based on the least populated segments of objective space. The leader selection mechanism considers the least crowded segments of the objective space and choses some non-dominated solutions as alpha, beta, or Delta. The selection is carried out using a roulette-wheel method with the following probability for each segment.

$$P_j = \frac{c}{N_j}$$  

(34)

Where $c$ is a constant and $N_j$ is the number of non-dominated particles in $j^{th}$ segment. The pseudo-code of the MOGWO is presented in the following.
Initialize the grey wolf population $X_i$ \((i = 1, 2, ..., n)\)

- Calculate the objective values for each search agent
- Find the non-dominated solutions and initialized the archive with them
- $X_a$ = SelectLeader(archive)
- Exclude alpha from the archive
- $X_b$ = SelectLeader(archive)
- Exclude beta from the archive
- $X_0$ = SelectLeader(archive)

\(t = 1;\)

\textbf{while} \((t < \text{Max number of iterations})\)

\textbf{for} each search agent

- Update the position of the current search agent by equations \((3.5)-(3.11)\)

\textbf{endfor}

- Update $a$, $A$, and $C$
- Initialize the grey wolf population $X_i$ \((i = 1, 2, ..., n)\)
- Calculate the objective values of all search agents
- Find the non-dominated solutions
- Update the archive with respect to the obtained non-dominated solutions

\textbf{if} the archive is full

- Run the grid mechanism to omit one of the current archive members
- Add the new solution to the archive

\textbf{endif}

\textbf{if} any of the new added solutions to the archive is located outside the hypercubes

- Update the grids to cover the new solution(s)

\textbf{endif}

\(X_a = \text{SelectLeader}(\text{archive})\)
- Exclude alpha from the archive
- $X_b$ = SelectLeader(archive)
- Exclude beta from the archive
- $X_0$ = SelectLeader(archive)

\(t = t + 1;\)

\textbf{endwhile}

return archive

4. Performance Evaluation

In this section, various test problems are solved in different sizes to evaluate the performance of the solution methodologies. In order to compare the solution methods, four performance measures are considered. The first measure is diversity. This measure, introduced by Zitzeler et al. (2001) and Zitzler and Thiele (1999), takes the measurement of the space spread by Pareto-optimal solutions; this measure shows the coverage of an algorithm. The high value of $D$ is desirable. The second measure is the Number of Non-dominated Pareto solutions (NNS). This measure determines the number of non-dominated Pareto-optimal solutions provided by each algorithm. The high value of NNS is desirable. The third measure is Hyper volume (HV) proposed by Zitzler and Thiele (1998). This measure determines the convergence of an algorithm. A significant value of HV is desirable. The fourth measure is CPU-Time. This measure shows the CPU time required by an algorithm to obtain Pareto-optimal solutions (Zitzler, 1999; Khalilpourazari et al., 2020; Fazli-Khalaf et al., 2017).

In order to compare the efficiency of the algorithms, the performance of the algorithms is evaluated, considering the four afore-mentioned performance measures. As suggested in the literature of metaheuristics, it is better to evaluate the performance of metaheuristics in small-to-medium and large scale separately (Khalilpourazari et al., 2019).

4.1 Small-to-medium size test problems

In this section, various small-to-medium size test problems are considered to evaluate the performance of the algorithms. For this purpose, twenty test problems with different sizes are considered. Then, each test problem is solved several times using each algorithm, and the average value of $D$ and HV measures are reported. Table 1 presents the specific details of the results.

<table>
<thead>
<tr>
<th>Size k-m-n</th>
<th>MOGWO $D_{\text{Average}}$</th>
<th>MOGWO $HV_{\text{Average}}$</th>
<th>MOWCA $D_{\text{Average}}$</th>
<th>MOWCA $HV_{\text{Average}}$</th>
<th>NSGA-II $D_{\text{Average}}$</th>
<th>NSGA-II $HV_{\text{Average}}$</th>
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<td>28765885.3</td>
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<td>9490871808</td>
</tr>
</tbody>
</table>

Table 1: Results of optimization in small-to-medium size test problems
As can be seen from the results, the performance of the algorithms in obtaining better Pareto frontier in terms of both coverage and convergence is very competitive. However, MOGWO with higher average diversity and hypervolume performs better than other algorithms. Therefore, to compare the algorithms, more analyses are needed. To show a schematic view of the results, figures 2-3 are presented that show Diversity and hypervolume measures obtained by each algorithm in each test problem.

Based on the results, MOGWO is the best algorithm in terms of diversity and hypervolume measures. To show how MOGWO outperforms other algorithms in diversity and hypervolume, a schematic view of Pareto solutions provided by each algorithm in each test problem is presented in figure 4.
The results ensure that not only MOGWO is able to converge to true Pareto frontier, but also, it provides Pareto solutions that are highly distributed among objectives and provide various alternatives for the decision-maker. Detailed information about the performance of the algorithms on NNS and CPU-Time measure are provided in Table 2.

Table 2
Performance of the algorithms on NNS and CPU-Time measure in small-to-medium test problems

<table>
<thead>
<tr>
<th>size k-m-n</th>
<th>MOGWO NNS</th>
<th>MOGWO CPU-Time (s)</th>
<th>MOGWO NNS</th>
<th>MOGWO CPU-Time (s)</th>
<th>MOGWO NNS</th>
<th>MOGWO CPU-Time (s)</th>
<th>MOGWO NNS</th>
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<th>MOGWO CPU-Time (s)</th>
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<th>MOGWO CPU-Time (s)</th>
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</table>
Based on the results, the algorithms are very competitive. To present a graphical representation of the results, Figures 5-6 are provided, which show detailed information on the performance of the algorithms on NNS and CPU_Time measures.

![Number of Non-dominated Pareto solutions provided by the algorithms](image1)

**Fig. 5. Number of Non-dominated Pareto solutions provided by the algorithms**

![CPU-Time of the algorithms in small-to-medium size test problems](image2)

**Fig. 6. CPU-Time of the algorithms in small-to-medium size test problems**

As is clear from the results, the NSGA-II and MOGWO perform better in providing a higher number of non-dominated solutions in each test problem. However, NSGA-II has a slightly higher average in NNS. On the other hand, MOGWO performs very well in computational time measures. The MOGWO can solve the problems faster in a specific number of function evaluations.

### 4.2 Large size test problems

In this section, various significant size test problems are generated in order to evaluate the performance of the algorithms in solving large size test instances. For this purpose, twenty test problems with different sizes are first generated. Then, each test problem is solved several times using each algorithm, and the average value of D and HV measures are reported. Table 3 presents the specific details of the results.
Fariba Maadanpour Safari and et al./Modeling and optimization of a tri-objective ...

Table 3
Results of optimization in large size test problems

<table>
<thead>
<tr>
<th>Size</th>
<th>MOGWO</th>
<th>MOWCA</th>
<th>MOPSO</th>
<th>NSGA-II</th>
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<td>HV_Average</td>
<td>HV_Average</td>
<td>HV_Average</td>
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<td>2.52241E+11</td>
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</table>

From the results, it becomes evident that the performance of the proposed algorithms in both coverage and convergence is competitive. Although MOGWO with higher average diversity and hypervolume performs better than other algorithms, more analyses still needed to make a reliable conclusion. To show a schematic view of the results, figures 7-8 are presented.
As is clear from figures 7-8, the MOGWO performs significantly better than other algorithms in obtaining efficient solutions with better convergence and higher coverage. To show detailed information about diversity and hypervolume, a schematic view of pareto solutions provided by each algorithm in each test problem is presented in figure 9.

![Fig. 9. A schematic view of pareto solutions provided by each algorithm in large size test problems](image)

It is also essential to compare the performance of the algorithms in other strong performance measures such as NNS and CPU-Time. Detailed information about performance of the algorithms on NNS and CPU-Time measure are provided in Table 4.

![Fig. 10. Number of Non-dominated Pareto solutions provided by the algorithms in large sizes](image)
Also, a graphical representation of the results is presented in Figures 10-11 which show detailed information about performance of the algorithms on NNS and CPU-Time measures.

From the results, the NSGA-II and MOGWO perform better in providing a higher number of non-dominated solutions in each test problem. Considering coverage and convergence measures (D and HV), the MOGWO performs significantly better than other algorithms and outperforms other algorithms in these measures. NSGA-II has a slightly higher average in NNS and CPU-Time. Since the algorithms are very competitive and in different performance measures, different algorithms are superior, a Multi-Criteria Decision Making (MCDM) approach is utilized to make the final decision about the superiority of the algorithms.

4.3. Determining the best solution method
The technique for order of Preference by Similarity to Ideal Solution (TOPSIS) method was first proposed by Hwang and Yoon (1981). The concept of the TOPSIS method is based on the selection of an alternative, which has the longest (shortest) distance from the negative (positive) ideal solution. In this section, the TOPSIS method is applied to determine the best algorithm in solving the problem. Since TOPSIS methods require predetermined weights for measures, equal weights for all four performance measures are considered in this research. To implement the TOPSIS method, the decision matrices for small and large sizes are presented as in Table 5.
To perform TOPSIS, first it is needed to normalize the decision matrix using Euclidean Norm:

\[ n_{ij} = \sqrt{\sum_{j} r_{ij}^{2}} \]  

(35)

Where \( r \) is the decision matrix and \( n \) is normalized decision matrix using Euclidean Norm. \( i \) is the index of algorithms and \( j \) is the measure index. To obtain weighted normalized decision matrix, Weight \( w \) (weight of performance measures) should multiply by normalized decision matrix.

\[
\text{weighted normalized matrix} = [v_{i}]_{m \times n} \cdot v_{j} = \text{Weight} \times n_{ij}
\]

(36)

Therefore, we can determine the ideal positive solution and the ideal negative solution as following:

\[
\text{idealsolution}^{+} = \{\max, v_{ij} : j \in j^{+}, \min, v_{ij} : j \in j^{-}\}
\]

(37)

\[
\text{idealsolution}^{-} = \{\max, v_{ij} : j \in j^{-}, \min, v_{ij} : j \in j^{+}\}
\]

(38)

Distance from the positive and negative ideal solutions for each algorithm is calculated using below formulas:

\[
d_{i}^{+} = \sqrt{\sum_{j=1}^{n} (v_{ij} - \text{idealsolution}^{+})}
\]

(39)

\[
d_{i}^{-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - \text{idealsolution}^{-})}
\]

(40)

Equation (41) presents the Similarity ratio formula which is calculated using \( d_{i}^{+} \) and \( d_{i}^{-} \).

\[
S_{i}^{+} = \frac{d_{i}^{-}}{d_{i}^{+} + d_{i}^{-}}
\]

(41)

The results of TOPSIS method are presented in Table 6.

Based on the results, MOGWO with higher similarity ratio is ranked as the first and best algorithm in solving the proposed tri-objective mathematical model. The results indicate that the MOGWO has a significantly higher similarity ratio comparing to other algorithms that guarantee its superiority. The NSGA-II is ranked second based on the values of similarity ratio, and MOPSO and MOWCA ranked third and fourth, respectively.

5. Conclusion and Future Research Directions

In this research, a tri-objective mathematical model was proposed for the Transportation-Location-Routing problem. The model considered a three-echelon supply chain and aimed to minimize total costs, maximize the minimum reliability of routes and establish a well-balanced set of routes. In order to demonstrate the efficiency of the mathematical model on one hand, and determination of a practical solution approach on the other hand, four metaheuristics including Multi-Objective Grey Wolf Optimizer (MOGWO), Multi-Objective Water Cycle Algorithm (MOWCA), Multi-objective Particle Swarm Optimization (MOPSO) and Non-Dominated Sorting Genetic Algorithm- II (NSGA-II) were utilized. The performance of the algorithms was evaluated in solving various test problems. The results revealed that the NSGA-II and MOGWO perform better in providing a greater number of non-dominated solutions in each test problem. Considering coverage and convergence measures (D and HV), the MOGWO performed significantly better than other algorithms and outperformed other algorithms in these measures. NSGA-II has a slightly higher average in NNS and CPU_Time. In the end, considering different performance measures including Diversity, hypervolume, Number of Non-dominated solutions, and CPU_Time, the best algorithm was determined by Technique for Order of Preference by
Similarity to Ideal Solution method. Based on the results, MOGWO with a higher similarity ratio was ranked as the first and best algorithm in solving the proposed tri-objective mathematical model. The results indicate that the MOGWO had a significantly higher similarity ratio comparing to other algorithms, which guarantees its superiority. The NSGA-II was ranked second based on the values of similarity ratio, and MOPSO and MOWCA were ranked third and fourth, respectively. The presented methodology can be applied to solve any location routing problems in real-world applications.

For future studies, it will be worthwhile to consider pickup and delivery decisions in the proposed model. This can significantly increase the complexity of the proposed model. Therefore, developing efficient solution techniques is essential. Considering uncertainty in the parameters of the mathematical model is another extension of this work since, in real-world applications, most of the main parameters are uncertain.

Conflict of interest: The authors declare that they have no conflict of interest.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

References


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http://www.qjie.ir/article_676273.html

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