Journal of Optimization in Industrial Engineering
Vol.15, Issue 1, Winter & Spring 2022, 145-160
DOI: 10.22094/JOIE.2021.761.1485

Research Article

A New Analysis of Critical Paths in Mega Projects with Interval Type-2 Fuzzy Activities By Considering Time, Cost, Risk, Quality, and Safety Factors

Yahya Dorfeshan [a], Seyed Meysam Mousavi [a,*], Behnam Vahdani [b]

[a] *Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran*
[b] *Department of Industrial Engineering, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*
Received 28 April 2017; Revised 06 October 2021; Accepted 10 October 2021

Abstract

Critical path method (CPM) is categorized as a popular tool for scheduling mega projects. In this paper, to enjoy the advantages of interval type-2 fuzzy sets (IT2FSs) and better address uncertainty for the activities' attributes, a new analysis model is presented to determine the critical path under an IT2F-environment. Also, new efficient factors on specifying critical paths, such as time, cost, risk, safety, and quality (TCRSQ), are presented to achieve a more robust plan assisting in megaproject success. Moreover, an IT2F weighting approach is suggested for specifying the weights of TCRSQ factors. Furthermore, a new IT2F-approach employing the relative preference relation is expressed for identifying the importance of each expert. Consequently, a new model for critical path determination procedure by considering efficient factors is developed under the IT2FSs environment. Finally, to demonstrate the suggested model's capability and the calculation process, an application from the previous research is solved. sm.mousavi@shahed.ac.ir

Keywords: Analysis of critical paths; Megaprojects; Interval type-2 fuzzy sets (IT2FSs); Risk, Quality and safety factors

1. Introduction

The critical path method (CPM) has been applied to manifold management problems for mega projects' planning as a useful tool (Chen and Hsueh, 2008). The longest path of projects from the point of time view is called the critical path. Each activity has some predecessor activities, and it cannot be started unless the predecessor activities are done. One-day delay on each activity of the critical path of megaprojects delays the entire project. CPM helps the managers in finding the crucial activities of megaprojects. To reduce the completion time of mega projects, it is necessary to centralized resources on the critical path's activities. Notably, CPM is assumed that the time of activities is deterministic and exact (Mehlawat and Gupta, 2016); in real-world situations, obtaining exact time and estimating resource consumptions for each activity is difficult. Further, in the planning phase of mega projects, considering the precise time of activities is challenging.

For solving this problem, the program evaluation and review technique (PERT) was extended to solve the critical path determination problems with incomplete and ambiguous data (Chen, 2007). The PERT that is proposed by Hillier and Liebermann (2001) and Krajewski and Ritzman (2005) and Monte Carlo simulation by Kurihara and Nishiuchi (2002) can be applied for critical path determination. The PERT employs various distributions to approximate the duration of activities (Mon et al., 1995). These various distributions are obtained based on the observation of the past performance of activities.

In PERT, three-time is estimated for each activity: a most possible (or most probable) time, a pessimistic time, and an optimistic time. These estimated three times are gathered from experts (Cristobal, 2012). While there are activities without past performance observation, then PERT is not helpful. For estimating the distributions of activities, the activities and their relationships should be recognized according to the work breakdown structure (WBS). Afterward, the distributions of times for the activities are estimated based on historical data. However, in many real situations, historical data are unavailable (Amiri et al., 2010; Foroozesh et al., 2017,2018,2019; Gitinavard et al., 2017a,b). Moreover, experts' subjective opinions are different from each other, which causes such probabilistic distributions fairly invalid (Ock and Han, 2010). Besides, some researchers have criticized PERT's

*Corresponding author Email address: sm.mousavi@shahed.ac.ir

method for optimistic results because of underestimating the pessimistic time (Nasir *et al.*, 2003). In such situations, fuzzy sets theory is introduced by Zadeh (1965) because it can tackle ambiguous input data, including feelings employing subjective opinions of experts, and does not need subsequent frequency distributions. Numerous studies have applied fuzzy sets theory for engineering and management cases (e.g., Vahdani et al., 2010,2011,2012,2014; Mohagheghi et al., 2015,2017; Mousavi et al., 2013,2014,2019; Mousavi & Vahdani, 2016).

For finding fuzzy critical paths, many methods were proposed over the past years. Kaur and Kumar (2014) have expressed a linear programming approach to determine the critical path with JMD representation of LR flat fuzzy numbers. Madhuri et al. (2012) introduced a method to specify the critical path utilizing L-L fuzzy numbers. Liang et al. (2004) expressed the fuzzy multi-objective model to obtain a critical project management path. Zareei et al. (2011) presented a fuzzy critical path selection method by assessing events. Chanas and Zielinski (2001) introduced a critical path method with considering activity times as fuzzy numbers. Zammori et al. (2009) expressed a new fuzzy MCDM approach for evaluating the critical path by considering various factors. Cristobal (2012) suggested an MCDM method (i.e., PROMETHEE) to determine the critical path under efficient factors. Amiri and Golozari (2011) chose the fuzzy critical path in networks employing time, cost, quality, and risk factors by the TOPSIS method. Mehlawat and Gupta (2015) introduced a fuzzy group MCDM method to specify the critical path. In the recent essential research of path area, conventional fuzzy sets have been used (Dorfeshan et al., 2019), whereas type-2 fuzzy set (T2FS) is a more powerful tool than classic fuzzy sets on reflecting uncertainty of real-world mega projects.

The meaning of T2FS was first introduced by (Zadeh 1975). The membership functions are two-dimensional in type-1 fuzzy sets, whereas in type- 2 fuzzy sets, the membership functions are three-dimensional. By providing extra freedom degrees, this new third-dimension of T2FSs enables it to directly model uncertainties. (Zhang and Zhang, 2013). In fact, compared to memberships of type-1 fuzzy sets which are crisp numbers in a type-2 fuzzy set, the memberships are type 1 fuzzy sets. Nevertheless, there are more uncertainty values in T2FSs than in type-1 fuzzy sets. In the general T2FSs, calculation complexity is very high; that is why interval T2FSs were presented by Mendel et al. (2006), which are the most broadly used type- 2 fuzzy sets applied to many other practical and real-world fields. (e.g., Liang and Mendel, 2000; Mendel and Wu, 2006; Castro et al., 2009; Jammeh et al., 2009).

Regarding the IT2FSs, Shukla and Muhuri (2019) utilized the IT2FSs for uncertainty considerations to select the travel time. Dorfeshan and Mousavi (2020) explained the relative preference relation on the IT2F-conditions for aircraft maintenance planning problems. Kiracı and Akan (2020) used the TOPSIS and AHP benefits simultaneously based on the IT2FSs for aircraft evaluation problems. Yılmaz and Kabak (2020) prioritized the distribution centers according to the IT2FSs for humanitarian logistics. Liu and Gao (2020) selected the best supplier based on the green paradigm on the IT2F-conditions using the Choquet Bonferroni means. Dorfeshan et al. (2021) enhanced the ARAS method for engineering project problems with IT2F situations.

To conclude from the above, given the advantages of T2FSs, applying them in a mega project environment is a practical model. Also, efficient factors like time, cost, risk, quality, and safety (TCRSQ) will be assumed as IT2FSs to specify the critical path in a network of projects. However, a new analysis model is presented for the critical paths regarding a relative preference relation (RPR) to determine each decision maker's weight and a new approach for calculating each efficient factor's importance. Finally, the critical path is obtained under TCRSQ criteria for planning and scheduling the mega projects through IT2F-environment. The following is presented to demonstrate the innovations of this research:

- IT2FSs are used in critical path analysis of mega projects to better address the uncertainty.
- A new method to determine each decision maker's weight is introduced by using RPR developed by IT2FSs.
- A new extension of the analysis model under an IT2F-environment is presented for the critical path analysis.
- A new development of the entropy method under the IT2F-environment is expressed to specify the weight of several important factors, such as TCRSQ.

The article continues as follows. Section 2 presents the fundamental knowledge and definitions of IT2FSs. Section 3 introduces the proposed model for critical path analysis. In Section 4, the application of the introduced model is presented. Lastly, Section 5 concludes the paper.

2. Type-2 Fuzzy Sets

The theory of type-1 fuzzy sets was proposed by Zadeh (1965). In type-1 fuzzy sets, the membership value is a crisp number in the range of 0 to 1. But, T2FSs are the extension of type-1, in which membership values are type-1 fuzzy sets. In the general T2FSs, calculation complexity is very high; that is why interval T2FSs are presented by Mendel et al. (2006). Recently, the T2FSs have been applied to the area of project management with interesting results (e.g., Moradi et al., 2018; Mohagheghi & Mousavi, 2019; Haghighi et al., 2019; Eshghi et al., 2019). The required fundamental knowledge of interval T2FSs is presented below.

$$\tilde{\tilde{Q}} = \int_{p\in P}\int_{\eta\in L_p} \mu_{\tilde{\tilde{Q}}}(p,\eta)/(p,\eta) \qquad (1)$$

where $L_p \subseteq [0,1]$ and $\iint$ displays union on all permissible p and η. $\tilde{\tilde{Q}}$ as a particular type of a T2FS that called interval T2FS is defined as follows:

$$\tilde{\tilde{Q}} = \int_{p\in P}\int_{\eta\in L_p} 1/(p,\eta) \qquad (2)$$

where $L_p \subseteq [0,1]$.

In a T2FS, the upper and lower membership functions are type-1 membership functions (Mendel et al., 2006).

A trapezoidal IT2FS is demonstrated as $\tilde{\tilde{A}}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u; H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)))$ where $\tilde{A}_i^L$ and $\tilde{A}_i^U$ are in fact type-1 fuzzy sets. $a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u, a_{i1}^L, a_{i2}^L, a_{i3}^L$ *and* a_{i4}^L are the reference points of the interval T2FS $\tilde{\tilde{A}}_i$, $H_\vartheta(\tilde{A}_i)$; displays the membership amount of the $a_{\vartheta(\vartheta+1)}^U$ in the upper trapezoidal membership grade $\tilde{A}_i^U, 1 \le \vartheta \le 2, H_\vartheta(A_i^L)$ indicates the membership amount of the $a_{\vartheta(\vartheta+1)}^L$ in the lower trapezoidal membership grade $\tilde{A}_i^L$, $H_1(\tilde{A}_i^U) \in [0, 1]$, $H_2(\tilde{A}_i^U) \in [0, 1]$, $H_1(\tilde{A}_i^L) \in [0, 1]$, $H_2(\tilde{A}_i^L) \in [0, 1]$, $1 \le \vartheta \le 2$, $1 \le i \le n$ (Chen and Lee, 2010). The membership function of a trapezoidal IT2FS is depicted in Fig. 1.

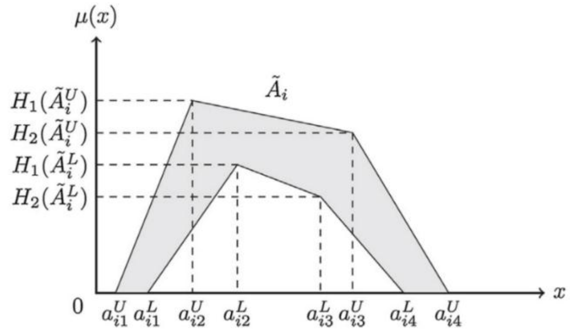

Fig. 1. A trapezoidal interval T2FS membership function (Chen and Lee, 2010)

The following are the basic algebraic operations of trapezoidal IT2FSs (Chen and Lee, 2010):

$$\begin{aligned}\tilde{\tilde{Q}}_1 &= (\tilde{Q}_1^U, \tilde{Q}_1^L) = ((q_{11}^u, q_{12}^u, q_{13}^u, q_{14}^u; \lambda_1(\tilde{Q}_1^U),\\ &\lambda_2(\tilde{Q}_1^U)), (q_{11}^L, q_{12}^L, q_{13}^L, q_{14}^L; \lambda_1(\tilde{Q}_1^L), \lambda_2(\tilde{Q}_1^L)))\\ \tilde{\tilde{Q}}_2 &= (\tilde{Q}_2^U, \tilde{Q}_2^L) = ((q_{21}^u, q_{22}^u, q_{23}^u, q_{24}^u; \lambda_1(\tilde{Q}_2^U),\\ &\lambda_2(\tilde{Q}_2^U)), (q_{21}^L, q_{22}^L, q_{23}^L, q_{24}^L; \lambda_1(\tilde{Q}_2^L), \lambda_2(\tilde{Q}_2^L)))\end{aligned} \qquad (3)$$

The addition operation:

$$\begin{aligned}&\tilde{\tilde{Q}}_1 \oplus \tilde{\tilde{Q}}_2 = (\tilde{Q}_1^U, \tilde{Q}_1^L) + (\tilde{Q}_2^U, \tilde{Q}_2^L) = ((q_{11}^u +\\ &q_{21}^u, q_{12}^u + q_{22}^u, q_{13}^u + q_{23}^u, q_{14}^u + q_{24}^u; \min(\lambda_1(\tilde{Q}_1^U),\\ &\lambda_1(\tilde{Q}_2^U)), \min(\lambda_2(\tilde{Q}_1^U), \lambda_2(\tilde{Q}_2^U))), q_{11}^L + q_{21}^L,\\ &q_{12}^L + q_{22}^L, q_{13}^L + q_{23}^L, q_{14}^L + q_{24}^L; \min(\lambda_1(\tilde{Q}_1^L),\\ &\lambda_1(\tilde{Q}_2^L)), \min(\lambda_2(\tilde{Q}_1^L), \lambda_2(\tilde{Q}_2^L)))).\end{aligned} \qquad (3)$$

The subtraction operation:

$$\begin{aligned}&\tilde{\tilde{Q}}_1 \Theta \tilde{\tilde{Q}}_2 = (\tilde{Q}_1^U, \tilde{Q}_1^L) - (\tilde{Q}_2^U, \tilde{Q}_2^L) = ((q_{11}^u - q_{24}^u,\\ &q_{12}^u - q_{23}^u, q_{13}^u - q_{22}^u, q_{14}^u - q_{21}^u; \min(\lambda_1(\tilde{Q}_1^U),\\ &\lambda_1(\tilde{Q}_2^U)), \min(\lambda_2(\tilde{Q}_1^U), \lambda_2(\tilde{Q}_2^U))), q_{11}^L - q_{24}^L,\\ &q_{12}^L - q_{23}^L, q_{13}^L - q_{22}^L, q_{14}^L - q_{21}^L; \min(\lambda_1(\tilde{Q}_1^L),\\ &\lambda_1(\tilde{Q}_2^L)), \min(\lambda_2(\tilde{Q}_1^L), \lambda_2(\tilde{Q}_2^L)))).\end{aligned} \qquad (4)$$

The multiplication operation:

$$\begin{aligned}&\tilde{\tilde{Q}}_1 \times \tilde{\tilde{Q}}_2 = (\tilde{Q}_1^U, \tilde{Q}_1^L) \times (\tilde{Q}_2^U, \tilde{Q}_2^L) = ((q_{11}^u \times q_{21}^u\\ &, q_{12}^u \times q_{22}^u, q_{13}^u \times q_{23}^u, q_{14}^u \times q_{24}^u; \min(\lambda_1(\tilde{Q}_1^U),\\ &\lambda_1(\tilde{Q}_2^U)), \min(\lambda_2(\tilde{Q}_1^U), \lambda_2(\tilde{Q}_2^U))), q_{11}^L \times q_{21}^L,\\ &q_{12}^L \times q_{22}^L, q_{13}^L \times q_{23}^L, q_{14}^L \times q_{24}^L; \min(\lambda_1(\tilde{Q}_1^L),\\ &\lambda_1(\tilde{Q}_2^L)), \min(\lambda_2(\tilde{Q}_1^L), \lambda_2(\tilde{Q}_2^L)))).\end{aligned} \qquad (5)$$

The multiplication with a scalar $r \geq 0$ (Sari and Kahraman, 2015):

$$r\tilde{\tilde{Q}}_1 = r(\tilde{Q}_1^U, \tilde{Q}_1^L) = ((r \times q_{11}^u, r \times q_{12}^u, r \times q_{13}^u, r \times q_{14}^u; \lambda_1(\tilde{Q}_1^U), \lambda_2(\tilde{Q}_1^U)), (r \times q_{11}^L, r \times q_{12}^L, r \times q_{13}^L, r \times q_{14}^L; \lambda_1(\tilde{Q}_1^L), \lambda_2(\tilde{Q}_1^L))) \quad (6)$$

The division with a scalar $r > 0$ (Sari and Kahraman, 2015):

$$\frac{\tilde{\tilde{Q}}_1}{r} = (\tilde{Q}_1^U, \tilde{Q}_1^L) = ((\frac{1}{r} \times q_{11}^u, \frac{1}{r} \times q_{12}^u, \frac{1}{r} \times q_{13}^u, \frac{1}{r} \times q_{14}^u; \lambda_1(\tilde{Q}_1^U), \lambda_2(\tilde{Q}_1^U)), (\frac{1}{r} \times q_{11}^L, \frac{1}{r} \times q_{12}^L, \frac{1}{r} \times q_{13}^L, \frac{1}{r} \times q_{14}^L; \lambda_1(\tilde{Q}_1^L), \lambda_2(\tilde{Q}_1^L))) \quad (7)$$

Inverse of a trapezoidal IT2FN (Sari and Kahraman, 2015):

$$\frac{1}{\tilde{\tilde{Q}}_1} \cong ((\frac{1}{q_{14}^U}, \frac{1}{q_{13}^U}, \frac{1}{q_{12}^U}, \frac{1}{q_{11}^U}; \lambda_1(\tilde{Q}_1^U), \lambda_2(\tilde{Q}_1^U)), (\frac{1}{q_{14}^L}, \frac{1}{q_{13}^L}, \frac{1}{q_{12}^L}, \frac{1}{q_{11}^L}; \lambda_1(\tilde{Q}_1^L), \lambda_2(\tilde{Q}_1^L))) \quad (8)$$

The division operation (Sari and Kahraman, 2015):

$$\frac{\tilde{\tilde{Q}}_1}{\tilde{\tilde{Q}}_2} \cong ((\frac{q_{11}^u}{q_{24}^u}, \frac{q_{12}^u}{q_{23}^u}, \frac{q_{13}^u}{q_{22}^u}, \frac{q_{14}^u}{q_{21}^u}; \min(\lambda_1(\tilde{Q}_1^U), \lambda_1(\tilde{Q}_2^U)), \min(\lambda_2(\tilde{Q}_1^U), \lambda_2(\tilde{Q}_2^U))), (\frac{q_{11}^L}{q_{24}^L}, \frac{q_{12}^L}{q_{23}^L}, \frac{q_{13}^L}{q_{22}^L}, \frac{q_{14}^L}{q_{21}^L}; \min(\lambda_1(\tilde{Q}_1^L), \lambda_1(\tilde{Q}_2^L)), \min(\lambda_2(\tilde{Q}_1^L), \lambda_2(\tilde{Q}_2^L)))). \quad (9)$$

The distance among trapezoidal IT2FNs is calculated by using the following:

$$d(\tilde{\tilde{Q}}_1, \tilde{\tilde{Q}}_2) = \sqrt{\begin{aligned} &(q_{11}^U - q_{21}^U)^2 + (q_{12}^U(\lambda_1(\tilde{Q}_1^U) \\ &- q_{22}^U(\lambda_1(\tilde{Q}_2^U))^2 + (q_{13}^U(\lambda_2(\tilde{Q}_1^U) \\ &- q_{23}^U(\lambda_2(\tilde{Q}_2^U))^2 + (q_{14}^U - q_{24}^U)^2 + \\ &(q_{11}^L - q_{21}^L)^2 + (q_{12}^L(\lambda_1(\tilde{Q}_1^L) - \\ &q_{22}^L(\lambda_1(\tilde{Q}_2^L))^2 + (q_{13}^L(\lambda_2(\tilde{Q}_1^L) \\ &- q_{23}^L(\lambda_2(\tilde{Q}_2^L))^2 + (q_{14}^L - q_{24}^L)^2 \end{aligned}} \quad (10)$$

3. Proposed Analysis Model for Critical Paths in Mega Projects

In this section, to benefit the advantages of IT2FSs, a new analysis model is presented to specify the critical path by including important factors of TCRSQ for megaprojects under an IT2F-environment. In fact, the DM's opinions on ratings of efficient factors for each project activity are gathered; then, qualitative factors are transformed into equivalent IT2F numbers. Also, a new method to determine the weight of experts is developed utilizing an RPR on IT2F-environment and is added to a new method for aggregating DMs' opinions. However, the importance of efficient factors on specifying the critical path is obtained by the IT2F-entropy method. Furthermore, the proposed framework for the critical path analysis problem is illustrated in Figure 2.

Step 1. Gather DMs' opinions on grading TCRSQ factors and form initial decision matrixes by considering each project network path as alternatives. Each path is called CPM_i ,where $i= 1, 2,\dots,m$. $\tilde{\tilde{D}}_K$ show decision K^{th} matrix and K denote the number of DM that $K=1,2,\dots, L$.

Step 2. The normalized decision matrix is calculated by using the following:

If criterion j denotes benefit, the normalization operation would be as follows (Liao, 2015):

$$\tilde{\tilde{D}}_K = \left(\tilde{\tilde{D}}_{ij}^k\right)_{m \times n} = \begin{bmatrix} \tilde{\tilde{D}}_{cpm_1,time}^K & \tilde{\tilde{D}}_{cpm_1,\text{cost}}^K & \tilde{\tilde{D}}_{cpm_1,risk}^K & \tilde{\tilde{D}}_{cpm_1,quality}^K & \tilde{\tilde{D}}_{cpm_1,safety}^K \\ \tilde{\tilde{D}}_{cpm_2,time}^K & \tilde{\tilde{D}}_{cpm_2,\text{cost}}^K & \tilde{\tilde{D}}_{cpm_2,risk}^K & \tilde{\tilde{D}}_{cpm_2,quality}^K & \tilde{\tilde{D}}_{cpm_2,safety}^K \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{D}}_{cpm_{m-1},time}^K & \tilde{\tilde{D}}_{cpm_{m-1},\text{cost}}^K & \tilde{\tilde{D}}_{cpm_{m-1},risk}^K & \tilde{\tilde{D}}_{cpm_{m-1},quality}^K & \tilde{\tilde{D}}_{cpm_{m-1},safety}^K \\ \tilde{\tilde{D}}_{cpm_m,time}^K & \tilde{\tilde{D}}_{cpm_m,\text{cost}}^K & \tilde{\tilde{D}}_{cpm_m,risk}^K & \tilde{\tilde{D}}_{cpm_m,quality}^K & \tilde{\tilde{D}}_{cpm_m,safety}^K \end{bmatrix} \quad (11)$$

$$\tilde{\tilde{N}}_{ij}=\begin{bmatrix}\left(\dfrac{a^{u}_{ij1}-a^{u}_{i-}}{range_j},\dfrac{a^{u}_{ij2}-a^{u}_{i-}}{range_j},\dfrac{a^{u}_{ij3}-a^{u}_{i-}}{range_j},\dfrac{a^{u}_{ij4}-a^{u}_{i-}}{range_j};\lambda_1(\tilde{N}^{u}_{ij}),\lambda_2(\tilde{N}^{u}_{ij})\right),\\ \left(\dfrac{a^{L}_{ij1}-a^{u}_{i-}}{range_j},\dfrac{a^{L}_{ij2}-a^{u}_{i-}}{range_j},\dfrac{a^{L}_{ij3}-a^{u}_{i-}}{range_j},\dfrac{a^{L}_{ij4}-a^{u}_{i-}}{range_j};\lambda_1(\tilde{N}^{L}_{ij}),\lambda_2(\tilde{N}^{L}_{ij})\right)\end{bmatrix} \quad (12)$$

If the j property belongs to cost criteria, then this normalization operation should be used:

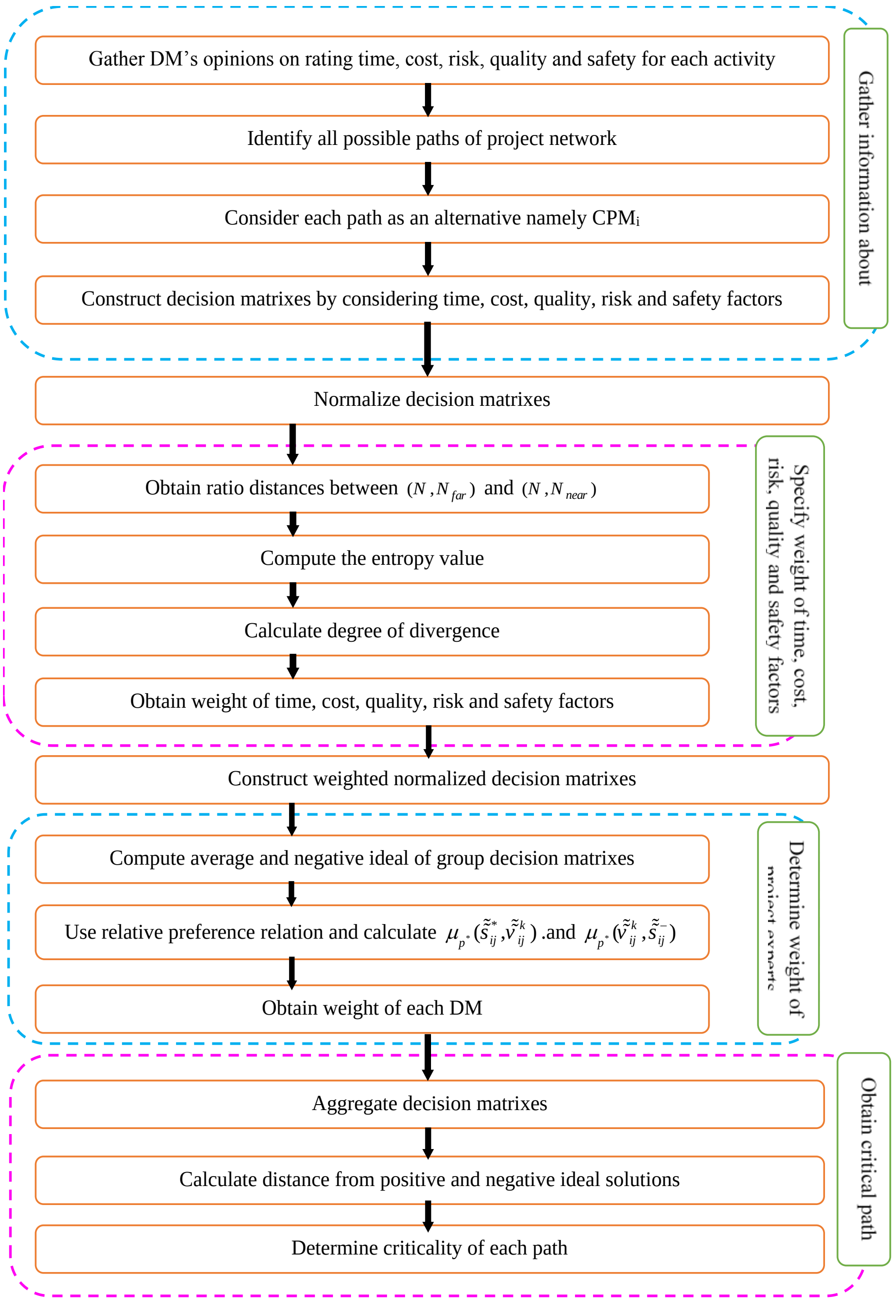

Fig. 2. Proposed framework to the critical path analysis

$$\tilde{\tilde{N}}_{ij}=\begin{bmatrix}\left(\begin{array}{l}\frac{a_{i+}^{u}-a_{ij4}^{u}}{range_{j}},\frac{a_{i+}^{u}-a_{ij3}^{u}}{range_{j}},\frac{a_{i+}^{u}-a_{ij2}^{u}}{range_{j}},\\ \frac{a_{i+}^{u}-a_{ij1}^{u}}{range_{j}};\lambda_1(\tilde{N}_{ij}^{u}),\lambda_2(\tilde{N}_{ij}^{u})\end{array}\right),\\ \left(\begin{array}{l}\frac{a_{i+}^{u}-a_{ij4}^{L}}{range_{j}},\frac{a_{i+}^{u}-a_{ij3}^{L}}{range_{j}},\frac{a_{i+}^{u}-a_{ij2}^{L}}{range_{j}},\\ \frac{a_{i+}^{u}-a_{ij1}^{L}}{range_{j}};\lambda_1(\tilde{N}_{ij}^{L}),\lambda_2(\tilde{N}_{ij}^{L})\end{array}\right)\end{bmatrix} \quad (13)$$

where $range_j=\max_i a_{ij4}^u-\min_i a_{ij1}^u$ let $a_{i+}^u=\max_i a_{ij4}^u$ and $a_{i-}^u=\min_i a_{ij1}^u$.

Step 3. A new approach is presented according to the ratio of distances between the (N, N_{near}) and (N, N_{far}) which is adapted based on Zamri and Abdullah (2013) to determine the weight of TCRSQ criteria.

Step 3-1. The value of T_j^k is computed as presented:

$$T_j^k=\frac{O_j^k}{\sum_{i=1}^{m}O_j^k} \quad (14)$$

Where

$$o_j^k=\frac{\sqrt{\sum_{j=1}^{n}\begin{bmatrix}(n_j^{k,near,L}-n_{1j}^k)^2+(n_j^{k,near,L}-n_{2j}^k)^2+\\ (n_j^{k,near,L}-n_{3j}^k)^2+(n_j^{k,near,L}-n_{4j}^k)^2+\\ (n_j^{k,near,U}-n_{1j}^k)^2+(n_j^{k,near,U}-n_{2j}^k)^2\\ +(n_j^{k,near,U}-n_{3j}^k)^2+(n_j^{k,near,U}-n_{4j}^k)^2\end{bmatrix}}}{\sqrt{\sum_{j=1}^{n}\begin{bmatrix}(n_j^{k,far,L}-n_{1j}^k)^2+(n_j^{k,far,L}-n_{2j}^k)^2+\\ (n_j^{k,far,L}-n_{3j}^k)^2+(n_j^{k,far,L}-n_{4j}^k)^2+\\ (n_j^{k,far,U}-n_{1j}^k)^2+(n_j^{k,far,U}-n_{2j}^k)^2+\\ (n_j^{k,far,U}-n_{3j}^k)^2+(n_j^{k,far,U}-n_{4j}^k)^2\end{bmatrix}}} \quad (15)$$

Step 3-2. The entropy value is calculated as follows:

$$EN_j^k=\left[-\beta\sum_{i=1}^{m}T_{ij}^k\ \ln T_{ij}^k\right] \quad (16)$$

β is a constant set as $(\ln(m))^{-1}$.

Step 3-3. The degree of divergence is computed as follows:

$$\varphi_j^k=\left[1-\left[-\beta\sum_{j=1}^{n}T_j^k\ \ln T_j^k\right]\right] \quad (17)$$

Step 3-4. The weight of several important factors, including TCRSQ, are calculated by using the following:

$$w_j^k=\frac{\left[1-\left[-\beta\sum_{j=1}^{n}T_j^k\ \ln T_j^k\right]\right]}{\sum_{j=1}^{n}\left[1-\left[-\beta\sum_{j=1}^{n}T_j^k\ \ln T_j^k\right]\right]} \quad (18)$$

Step 4. The weighted normalized decision matrix is computed to specify the critical path as follows:

$$V_P=(\tilde{\tilde{V}}_{ij}^{p})_{m\times n}=\begin{pmatrix}w_1^p\tilde{\tilde{n}}_{11}^p & \cdots & w_n^p\tilde{\tilde{n}}_{1n}^p\\ \vdots & \ddots & \vdots\\ w_1^p\tilde{\tilde{n}}_{m1}^p & \cdots & w_n^p\tilde{\tilde{n}}_{mn}^p\end{pmatrix} \quad (19)$$

Step 5. In this step, by using the extension of RPR on the IT2F-environment, each DM's weight is computed.

Step 5-1. The IT2F average of group decision matrixes (S*) and IT2F negative ideal solution (S-) matrixes are constructed based on Eqs. (27) and (28).

$$s^*=\left(\tilde{s}_{ij}^{*U},\tilde{s}_{ij}^{*L}\right)_{m\times n}=\begin{bmatrix}(\tilde{s}_{11}^{*U},\tilde{s}_{11}^{*L}) & (\tilde{s}_{12}^{*U},\tilde{s}_{12}^{*L}) & \cdots & (\tilde{s}_{1n}^{*U},\tilde{s}_{1n}^{*L})\\ (\tilde{s}_{21}^{*U},\tilde{s}_{21}^{*L}) & (\tilde{s}_{22}^{*U},\tilde{s}_{22}^{*L}) & \cdots & (\tilde{s}_{2n}^{*U},\tilde{s}_{2n}^{*L})\\ \vdots & \vdots & \ddots & \vdots\\ (\tilde{s}_{m1}^{*U},\tilde{s}_{m1}^{*L}) & (\tilde{s}_{m2}^{*U},\tilde{s}_{m2}^{*L}) & \cdots & (\tilde{s}_{mn}^{*U},\tilde{s}_{mn}^{*L})\end{bmatrix} \quad (20)$$

$$s^-=\left(\tilde{s}_{ij}^{-U},\tilde{s}_{ij}^{-L}\right)_{m\times n}=\begin{bmatrix}(\tilde{s}_{11}^{-U},\tilde{s}_{11}^{-L}) & (\tilde{s}_{12}^{-U},\tilde{s}_{12}^{-L}) & \cdots & (\tilde{s}_{1n}^{-U},\tilde{s}_{1n}^{-L})\\ (\tilde{s}_{21}^{-U},\tilde{s}_{21}^{-L}) & (\tilde{s}_{22}^{-U},\tilde{s}_{22}^{-L}) & \cdots & (\tilde{s}_{2n}^{-U},\tilde{s}_{2n}^{-L})\\ \vdots & \vdots & \ddots & \vdots\\ (\tilde{s}_{m1}^{-U},\tilde{s}_{m1}^{-L}) & (\tilde{s}_{m2}^{-U},\tilde{s}_{m2}^{-L}) & \cdots & (\tilde{s}_{mn}^{-U},\tilde{s}_{mn}^{-L})\end{bmatrix} \quad (21)$$

$$\tilde{s}_{ij}^{*U} = (\frac{1}{p}\sum_{p=1}^{k} v_{ij1}^{UP}, \frac{1}{p}\sum_{p=1}^{k} v_{ij2}^{UP}, \frac{1}{p}\sum_{p=1}^{k} v_{ij3}^{UP}, \frac{1}{p}\sum_{p=1}^{k} v_{ij4}^{UP}; \lambda_1(\tilde{S}_{ij}^{U}), \lambda_2(\tilde{S}_{ij}^{U}))$$

$$\tilde{s}_{ij}^{*L} = (\frac{1}{p}\sum_{p=1}^{k} v_{ij1}^{LP}, \frac{1}{p}\sum_{p=1}^{k} v_{ij2}^{LP}, \frac{1}{p}\sum_{p=1}^{k} v_{ij3}^{LP}, \frac{1}{p}\sum_{p=1}^{k} v_{ij4}^{LP}; \lambda_1(\tilde{S}_{ij}^{L}), \lambda_2(\tilde{S}_{ij}^{L})), \quad p = 1, 2, ..., k \tag{22}$$

$$\tilde{s}_{ij}^{-U} = (\max_p v_{ij1}^{UP}, \max_p v_{ij2}^{UP}, \max_p v_{ij3}^{UP}, \max_p v_{ij4}^{UP}; \lambda_1(\tilde{S}_{ij}^{U}), \lambda_2(\tilde{S}_{ij}^{U}))$$

$$\tilde{s}_{ij}^{-L} = (\min_p v_{ij1}^{LP}, \min_p v_{ij2}^{LP}, \min_p v_{ij3}^{LP}, \min_p v_{ij4}^{LP}; \lambda_1(\tilde{S}_{ij}^{L}), \lambda_2(\tilde{S}_{ij}^{L}))$$

Step 5-2. Calculating $\mu_{p^*}(\tilde{\tilde{s}}_{ij}^{*}, \tilde{\tilde{v}}_{ij}^{k})$ that denote RPR of $\tilde{\tilde{s}}_{ij}^{*}$ over each decision matrix $\tilde{\tilde{v}}_{ij}^{k}$ and $\mu_{p^*}(\tilde{\tilde{v}}_{ij}^{k}, \tilde{\tilde{s}}_{ij}^{-})$ that illustrate RPR of each decision matrix $\tilde{\tilde{v}}_{ij}^{k}$ over $\tilde{\tilde{s}}_{ij}^{-}$ based on the following relations.

$$\mu_{P^*}^{p}(\tilde{\tilde{s}}_{ij}^{*}, \tilde{\tilde{v}}_{ij}^{k}) = \frac{1}{2}\begin{bmatrix} \mu_{p^*}(\tilde{s}_{ij}^{*U}, \tilde{v}_{ij}^{pU}) + \\ \mu_{p^*}(\tilde{s}_{ij}^{*L}, \tilde{v}_{ij}^{pL}) \end{bmatrix} =$$

$$\frac{1}{2}\left[\frac{1}{2}\left(\frac{(\frac{1}{p}\sum_{p=1}^{k} v_{ij1}^{UP} - v_{ij4}^{pU}) + (\frac{1}{p}\sum_{p=1}^{k} v_{ij2}^{UP} - v_{ij3}^{pU}) + (\frac{1}{p}\sum_{p=1}^{k} v_{ij3}^{UP} - v_{ij2}^{pU}) + (\frac{1}{p}\sum_{p=1}^{k} v_{ij4}^{UP} - v_{ij1}^{pU})}{2\left\|M_{S_{ij}}^{pU}\right\|} + 1\right) + \frac{1}{2}\left(\frac{(\frac{1}{p}\sum_{p=1}^{k} v_{ij1}^{LP} - v_{ij4}^{pL}) + (\frac{1}{p}\sum_{p=1}^{k} v_{ij2}^{LP} - v_{ij3}^{pL}) + (\frac{1}{p}\sum_{p=1}^{k} v_{ij3}^{LP} - v_{ij2}^{pL}) + (\frac{1}{p}\sum_{p=1}^{k} v_{ij4}^{LP} - v_{ij1}^{pL})}{2\left\|M_{S_{ij}}^{pL}\right\|} + 1\right)\right] \tag{23}$$

Where

$$\left\|M_{S_{ij}}^{pU}\right\| = \begin{cases} \dfrac{(m_{sij1}^{+pU} - m_{sij4}^{-pU}) + (m_{sij2}^{+pU} - m_{sij3}^{-pU}) + (m_{sij3}^{+pU} - m_{sij2}^{-pU}) + (m_{sij4}^{+pU} - m_{sij1}^{-pU})}{2} & if \quad m_{sij1}^{+pU} \geq m_{sij4}^{-pU} \\ \dfrac{(m_{sij1}^{+pU} - m_{sij4}^{-pU}) + (m_{sij2}^{+pU} - m_{sij3}^{-pU}) + (m_{sij3}^{+pU} - m_{sij2}^{-pU}) + (m_{sij4}^{+pU} - m_{sij1}^{-pU})}{2} + 2(m_{sij4}^{-pU} - m_{sij1}^{+pU}) & if \quad m_{sij1}^{+pU} \prec m_{sij4}^{-pU} \end{cases} \tag{24}$$

$$\left\|M_{S_{ij}}^{pL}\right\| = \begin{cases} \dfrac{(m_{sij1}^{+pL} - m_{sij4}^{-pL}) + (m_{sij2}^{+pL} - m_{sij3}^{-pL}) + (m_{sij3}^{+pL} - m_{sij2}^{-pL}) + (m_{sij4}^{+pL} - m_{sij1}^{-pL})}{2} & if \quad m_{sij1}^{+pL} \geq m_{sij4}^{-pL} \\ \dfrac{(m_{sij1}^{+pL} - m_{sij4}^{-pL}) + (m_{sij2}^{+pL} - m_{sij3}^{-pL}) + (m_{sij3}^{+pL} - m_{sij2}^{-pL}) + (m_{sij4}^{+pL} - m_{sij1}^{-pL})}{2} + 2(m_{sij4}^{-pL} - m_{sij1}^{+pL}) & if \quad m_{sij1}^{+pL} \prec m_{sij4}^{-pL} \end{cases} \tag{25}$$

$$m_{sij1}^{+pL} = \max\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij1}^{LP}, v_{ij1}^{pL}\right\}, \quad m_{sij2}^{+pL} = \max\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij2}^{LP}, v_{ij2}^{pL}\right\}, \quad m_{sij3}^{+pL} = \max\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij3}^{LP}, v_{ij3}^{pL}\right\}, \quad m_{sij4}^{+pL} = \max\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij4}^{LP}, v_{ij4}^{pL}\right\} \tag{26}$$

$$
\begin{aligned}
m_{sij1}^{-pU} &= \min\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij1}^{UP}, v_{ij1}^{pU}\right\},\\
m_{sij2}^{-pU} &= \min\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij2}^{UP}, v_{ij2}^{pU}\right\},\\
m_{sij3}^{-pU} &= \min\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij3}^{UP}, v_{ij3}^{pU}\right\},\\
m_{sij4}^{-pU} &= \min\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij4}^{UP}, v_{ij4}^{pU}\right\}\\
m_{sij1}^{-pL} &= \min\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij1}^{LP}, v_{ij1}^{pL}\right\},\\
m_{sij2}^{-pL} &= \min\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij2}^{LP}, v_{ij2}^{pL}\right\},\\
m_{sij3}^{-pL} &= \min\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij3}^{LP}, v_{ij3}^{pL}\right\},\\
m_{sij4}^{-pL} &= \min\left\{\frac{1}{p}\sum_{p=1}^{k} v_{ij4}^{LP}, v_{ij4}^{pL}\right\}
\end{aligned}
\tag{27}
$$

and,

$$
\mu_{P^*}^{p}(\tilde{\tilde{v}}_{ij}^{k}, \tilde{\tilde{s}}_{ij}^{-}) = \frac{1}{2}\left[\mu_{p^*}(\tilde{v}_{ij}^{pU}, \tilde{s}_{ij}^{-U}) + \mu_{p^*}(\tilde{v}_{ij}^{pL}, \tilde{s}_{ij}^{-L})\right] =
$$

$$
\frac{1}{2}\left[\frac{1}{2}\left(\frac{(v_{ij1}^{pU} - \max_p v_{ij4}^{UP}) + (v_{ij2}^{pU} - \max_p v_{ij3}^{UP}) + (v_{ij3}^{kU} - \max_p v_{ij2}^{UP}) + (v_{ij4}^{pU} - \max_p v_{ij1}^{UP})}{2\left\|M_{S_{ij}}^{pU}\right\|} + 1\right) + \frac{1}{2}\left(\frac{(v_{ij1}^{pL} - \min_p v_{ij4}^{LP}) + (v_{ij2}^{pL} - \min_p v_{ij3}^{LP}) + (v_{ij3}^{pL} - \min_p v_{ij2}^{LP}) + (v_{ij4}^{pL} - \min_p v_{ij1}^{LP})}{2\left\|M_{S_{ij}}^{pL}\right\|} + 1\right)\right]
\tag{28}
$$

Where

$$
\left\|M_{S_{ij}}^{pU}\right\| = \begin{cases}
\dfrac{(m_{sij1}^{+pU} - m_{sij4}^{-pU}) + (m_{sij2}^{+pU} - m_{sij3}^{-pU}) + (m_{sij3}^{+pU} - m_{sij2}^{-pU}) + (m_{sij4}^{+pU} - m_{sij1}^{-pU})}{2} \\
\quad if \quad m_{sij1}^{+pU} \geq m_{sij4}^{-pU} \\
\dfrac{(m_{sij1}^{+pU} - m_{sij4}^{-pU}) + (m_{sij2}^{+pU} - m_{sij3}^{-pU}) + (m_{sij3}^{+pU} - m_{sij2}^{-pU}) + (m_{sij4}^{+pU} - m_{sij1}^{-pU})}{2} \\
\quad +2(m_{sij4}^{-pU} - m_{sij1}^{+pU}) \\
\quad if \quad m_{sij1}^{+pU} \prec m_{sij4}^{-pU}
\end{cases}
\tag{29}
$$

$$
\left\|M_{S_{ij}}^{kL}\right\| = \begin{cases}
\dfrac{(m_{sij1}^{+pL} - m_{sij4}^{-pL}) + (m_{sij2}^{+pL} - m_{sij3}^{-pL}) + (m_{sij3}^{+pL} - m_{sij2}^{-pL}) + (m_{sij4}^{+pL} - m_{sij1}^{-pL})}{2} \\
\quad if \quad m_{sij1}^{+kL} \geq m_{sij4}^{-kL} \\
\dfrac{(m_{sij1}^{+pL} - m_{sij4}^{-pL}) + (m_{sij2}^{+pL} - m_{sij3}^{-pL}) + (m_{sij3}^{+pL} - m_{sij2}^{-pL}) + (m_{sij4}^{+pL} - m_{sij1}^{-pL})}{2} \\
\quad +2(m_{sij4}^{-kL} - m_{sij1}^{+kL}) \\
\quad if \quad m_{sij1}^{+kL} \prec m_{sij4}^{-kL}
\end{cases}
\tag{30}
$$

$$
\begin{aligned}
m_{sij1}^{+pU} &= \max\left\{\max_p v_{ij1}^{UP}, v_{ij1}^{pU}\right\},\\
m_{sij2}^{+pU} &= \max\left\{\max_p v_{ij2}^{UP}, v_{ij2}^{pU}\right\},\\
m_{sij3}^{+pU} &= \max\left\{\max_p v_{ij3}^{UP}, v_{ij3}^{pU}\right\},\\
m_{sij4}^{+pU} &= \max\left\{\max_p v_{ij4}^{UP}, v_{ij4}^{pU}\right\}\\
m_{sij1}^{+pL} &= \max\left\{\min_p v_{ij1}^{UP}, v_{ij1}^{pU}\right\},\\
m_{sij2}^{+pL} &= \max\left\{\min_p v_{ij2}^{UP}, v_{ij2}^{pU}\right\},\\
m_{sij3}^{+pL} &= \max\left\{\min_p v_{ij3}^{UP}, v_{ij3}^{pU}\right\},\\
m_{sij4}^{+pL} &= \max\left\{\min_p v_{ij4}^{UP}, v_{ij4}^{pU}\right\}
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
m_{sij1}^{-pU} &= \min\left\{\max_p v_{ij1}^{UP}, v_{ij1}^{pU}\right\},\\
m_{sij2}^{-pU} &= \min\left\{\max_p v_{ij2}^{UP}, v_{ij2}^{pU}\right\},\\
m_{sij3}^{-pU} &= \min\left\{\max_p v_{ij3}^{UP}, v_{ij3}^{pU}\right\},\\
m_{sij4}^{-pU} &= \min\left\{\max_p v_{ij4}^{UP}, v_{ij4}^{pU}\right\}\\
m_{sij1}^{-pL} &= \min\left\{\min_p v_{ij1}^{UP}, v_{ij1}^{pU}\right\},\\
m_{sij2}^{-pL} &= \min\left\{\min_p v_{ij2}^{UP}, v_{ij2}^{pU}\right\},\\
m_{sij3}^{-pL} &= \min\left\{\min_p v_{ij3}^{UP}, v_{ij3}^{pU}\right\},\\
m_{sij4}^{-pL} &= \min\left\{\min_p v_{ij4}^{UP}, v_{ij4}^{pU}\right\}
\end{aligned} \tag{32}
$$

Step 5-3. By using the following, the relative closeness of each DM is determined:

$$
\Omega_p = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{P^*}^{p}(\tilde{\tilde{s}}_{ij}^{*}, \tilde{\tilde{v}}_{ij}^{p})}{\sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{P^*}^{p}(\tilde{\tilde{s}}_{ij}^{*}, \tilde{\tilde{v}}_{ij}^{p}) + \sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{P^*}^{p}(\tilde{\tilde{v}}_{ij}^{p}, \tilde{\tilde{s}}_{ij}^{-})}, \quad p = 1, 2, ..., k \tag{33}
$$

Step 5-4. The final weight (FW) of each DM is specified as follows:

$$
FW_p = \frac{\Omega_p}{\sum_{p=1}^{k}\Omega_p} \tag{34}
$$

Step 6. Aggregate DM's opinions to determine the critical path by using the following:

$$
\tilde{\tilde{\theta}} = \frac{\sum_{p=1}^{k} \begin{gathered}((FW_P * v_{ij}^{pU}, FW_P * v_{ij}^{pU}, FW_P * v_{ij}^{pU}, FW_P * v_{ij}^{pU}, \lambda_1(\tilde{v}_{ij}^{pU}), \lambda_2(\tilde{v}_{ij}^{pU})), (FW_P * v_{ij}^{pL},\\ FW_P * v_{ij}^{pL}, FW_P * v_{ij}^{pL}, FW_P * v_{ij}^{pL}, \lambda_1(\tilde{v}_{ij}^{pL}), \lambda_2(\tilde{v}_{ij}^{pL})))\end{gathered}}{\sum_{p=1}^{k} FW_P} \tag{35}
$$

Step 7. Positive ideal and negative ideal solutions are determined as follows:

$$
\begin{aligned}
&\omega^{+U} = (\tilde{\tilde{\theta}}_1^{+U}, \tilde{\tilde{\theta}}_2^{+U}, \tilde{\tilde{\theta}}_3^{+U}, ..., \tilde{\tilde{\theta}}_n^{+U}) = \\
&\left\{(\max_i \tilde{\tilde{\theta}}_{ij}^{U} \middle| j \in \nu), (\min_i \tilde{\tilde{\theta}}_{ij}^{U} \middle| j \in \gamma)\right\}\\
&\omega^{+L} = (\tilde{\tilde{\theta}}_1^{+L}, \tilde{\tilde{\theta}}_2^{+L}, \tilde{\tilde{\theta}}_3^{+L}, ..., \tilde{\tilde{\theta}}_n^{+L}) = \\
&\left\{(\max_i \tilde{\tilde{\theta}}_{ij}^{L} \middle| j \in \nu), (\min_i \tilde{\tilde{\theta}}_{ij}^{L} \middle| j \in \gamma)\right\}\\
&\omega^{-U} = (\tilde{\tilde{\theta}}_1^{-U}, \tilde{\tilde{\theta}}_2^{-U}, \tilde{\tilde{\theta}}_3^{-U}, ..., \tilde{\tilde{\theta}}_n^{-U}) = \\
&\left\{(\min \tilde{\tilde{\theta}}_{ij}^{U} \middle| j \in \nu), (\max_i \tilde{\tilde{\theta}}_{ij}^{U} \middle| j \in \gamma)\right\}\\
&\omega^{-L} = (\tilde{\tilde{\theta}}_1^{-L}, \tilde{\tilde{\theta}}_2^{-L}, \tilde{\tilde{\theta}}_3^{-L}, ..., \tilde{\tilde{\theta}}_n^{-L}) = \\
&\left\{(\min \tilde{\tilde{\theta}}_{ij}^{L} \middle| j \in \nu), (\max_i \tilde{\tilde{\theta}}_{ij}^{L} \middle| j \in \gamma)\right\}
\end{aligned} \tag{36}
$$

Where

$$
\begin{aligned}
&\max_i \tilde{\theta}_{ij}^{U} = (\max_i \theta_{ij1}^{U}, \max_i \theta_{ij2}^{U}, \max_i \theta_{ij3}^{U}, \max_i \theta_{ij4}^{U})\\
&\max_i \tilde{\theta}_{ij}^{L} = (\max_i \theta_{ij1}^{L}, \max_i \theta_{ij2}^{L}, \max_i \theta_{ij3}^{L}, \max_i \theta_{ij4}^{L})\\
&\min_i \tilde{\theta}_{ij}^{U} = (\min_i \theta_{ij1}^{U}, \min_i \theta_{ij2}^{U}, \min_i \theta_{ij3}^{U}, \min_i \theta_{ij4}^{U})\\
&\min_i \tilde{\theta}_{ij}^{L} = (\min_i \theta_{ij1}^{L}, \min_i \theta_{ij2}^{L}, \min_i \theta_{ij3}^{L}, \min_i \theta_{ij4}^{L})\\
&\nu = 1, 2, ..., n \in \text{ benefit criteria}\\
&\gamma = 1, 2, ..., n \in \text{ cost criteria}
\end{aligned} \tag{37}
$$

Step 8. By using the relations (38) and (39), the distance from positive ideal solution (D^+) and distance from negative ideal solution (D^-) are computed:

$$D^{+}=\sum_{j=1}^{n}\sqrt{\begin{array}{l}(G_{ij1}^{U}-\max_{i}G_{ij1}^{U})^{2}+(G_{ij2}^{U}\lambda_{1}(\tilde{G}_{1}^{U})\\-\max_{i}G_{ij2}^{U}\lambda_{1}(\tilde{G}_{2}^{U}))^{2}+(G_{ij3}^{U}\lambda_{2}(\tilde{G}_{1}^{U})\\-\max_{i}G_{ij3}^{U}\lambda_{2}(\tilde{G}_{2}^{U}))^{2}+(G_{ij4}^{U}-\max_{i}G_{ij4}^{U})^{2}\\+(G_{ij1}^{L}-\max_{i}G_{ij1}^{L})^{2}+(G_{ij2}^{L}\lambda_{1}(\tilde{G}_{1}^{L})\\-\max_{i}G_{ij2}^{L}\lambda_{1}(\tilde{G}_{2}^{L}))^{2}+(G_{ij3}^{L}\lambda_{2}(\tilde{G}_{1}^{L})\\-\max_{i}G_{ij3}^{L}\lambda_{2}(\tilde{G}_{2}^{L}))^{2}+(G_{ij4}^{L}-\max_{i}G_{ij4}^{L})^{2}\end{array}} \quad (38)$$

and,

$$D^{-}=\sum_{j=1}^{n}\sqrt{\begin{array}{l}(G_{ij1}^{U}-\min_{i}G_{ij1}^{U})^{2}+(G_{ij2}^{U}\lambda_{1}(\tilde{G}_{1}^{U})\\-\min_{i}G_{ij2}^{U}\lambda_{1}(\tilde{G}_{2}^{U}))^{2}+(G_{ij3}^{U}\lambda_{2}(\tilde{G}_{1}^{U})\\-\min_{i}G_{ij3}^{U}\lambda_{2}(\tilde{G}_{2}^{U}))^{2}+(G_{ij4}^{U}-\min_{i}G_{ij4}^{U})^{2}\\+(G_{ij1}^{L}-\min_{i}G_{ij1}^{L})^{2}+(G_{ij2}^{L}\lambda_{1}(\tilde{G}_{1}^{L})\\-\min_{i}G_{ij2}^{L}\lambda_{1}(\tilde{G}_{2}^{L}))^{2}+(G_{ij3}^{L}\lambda_{2}(\tilde{G}_{1}^{L})\\-\min_{i}G_{ij3}^{L}\lambda_{2}(\tilde{G}_{2}^{L}))^{2}+(G_{ij4}^{L}-\min_{i}G_{ij4}^{L})^{2}\end{array}} \quad (39)$$

Step 9. Calculating the final ranking (F) and determining the critical path by using the following:

$$F_i=\left(\frac{D_i^{-}}{D_{i,\max}^{-}}\right)+\left(\frac{D_i^{+}}{D_{i,\max}^{+}}\right),\ i=1,2,...,m \quad (40)$$

Rank the values of F_i in decreasing order of F_i.

4. Application

In this part, to better demonstrate the suggested analysis model's capability and applicability, an application example of the literature (Amiri and Golozari, 2011) is adopted and solved. To improve the planning phase of project and reach a reliable plan, a new model is presented for determining the critical path with taking the efficient criteria into account, e.g., TCRSQ. In Figure 3, the significant activity of the project is depicted. Also, to assess qualitative factors such as risk, quality, and safety, they are used as linguistic variables, and their IT2F-equivalent are shown in Table 1.

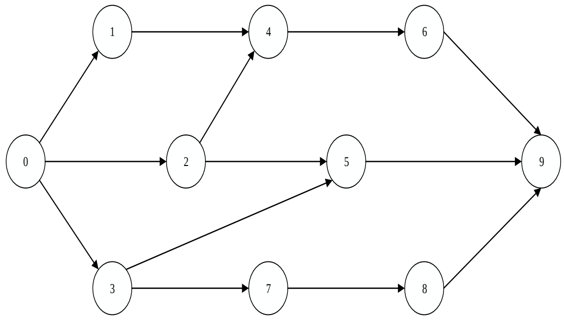

Fig. 3. Project network (Amiri and Golozari, 2011)

Table 1

Equivalent interval T2FSs of linguistic variables (Chen and Lee, 2010).

Interval T2FSs	Linguistic variables
((0.9,1,1,1;1,1), (0.95,1,1,1;0.9,0.9))	Very High (VH)
((0.7,0.9,0.9,1;1,1), (0.8,0.9,0.9,0.95;0.9,0.9))	High (H)
((0.5,0.7,0.7,0.9;1,1), (0.6,0.7,0.7,0.8;0.9,0.9))	Medium-High (MH)
((0.3,0.5,0.5,0.7;1,1), (0.4,0.5,0.5,0.6;0.9,0.9))	Medium (M)
((0.1,0.3,0.3,0.5;1,1), (0.2,0.3,0.3,0.4;0.9,0.9))	Medium Low (ML)
((0,0.1,0.1,0.3;1,1), (0.05,0.1,0.1,0.2;0.9,0.9))	Low (L)
((0,0,0,0.1;1,1), (0,0,0,0.05;0.9,0.9))	Very Low (VL)

Step 1. Here, a team of three experts is formed as DMs, and their views on ratings of TCRSQ criteria for each activity are gathered. This information is depicted in Tables 2-4.

Table 2

IT2F time of activities (Days)

ACT.	Experts		
	D_1	D_2	D_3
1-0	(3,5,8,10;1,1),(4,6,7,9;0.9,0.9)	(1,3,6,8;1,1),(2,4,5,7;0.9,0.9)	(3,5,8,10;1,1),(4,6,7,9;0.9,0.9)
2-0	(3,5,8,10;1,1),(4,6,7,9;0.9,0.9)	(3,5,8,10;1,1),(4,6,7,9;0.9,0.9)	(3,5,8,10;1,1),(4,6,7,9;0.9,0.9)
3-0	(7,9,12,14;1,1),(8,10,11,13.0.9,0.9)	(5,8,11,13;1,1),(6,9,10,12;0.9,0.9)	(6,10,14,18;1,1),(7,11,13,17,0.9,0.9)
4-1	(4,7,10,13;1,1),(5,8,9,12;0.9,0.9)	(3,5,8,10;1,1),(4,6,7,9;0.9,0.9)	(6,8,11,13;1,1),(7,9,10,12;0.9,0.9)
4-2	(9,12,15,18;1,1),(10,13,14,17;0.9,0.9)	(10,14,18,22;1,1),(11,15,17,21;0.9,0.9)	(10,12,15,17;1,1),(11,13,14,16;.9,.9)
5-3	(7,9,12,14;1,1),(8,10,11,13,.9,.9)	(8,10,13,15;1,1),(9,11,12,14;0.9,0.9)	(6,9,12,15;1,1),(7,10,11,14;0.9,0.9)
8-7	(14,17,20,23;1,1),(15,18,19,22;0.9,0.9)	(12,16,20,24;1,1),(13,17,19,23;0.9,0.9)	(13,15,18,20;1,1),(14,16,17,19;0.9,0.9)
5-2	(10,13,16,19;1,1),(11,14,15,18;0.9,0.9)	(11,14,17,20;1,1),(12,15,16,19;0.9,0.9)	(12,14,17,20;1,1),(13,15,16,19,0.9,0.9)
6-4	(9,14,19,24;1,1),(10,15,18,23;0.9,0.9)	(11,14,17,20;1,1),(12,15,16,19;0.9,0.9)	(10,14,18,22;1,1),(11,15,17,21;0.9,0.9)
7-3	(7,9,12,14;1,1),(8,10,11,13;0.9,0.9)	(5,7,10,12;1,1),(6,8,9,11,0.9,0.9)	(7,9,12,14;1,1),(8,10,11,13;0.9,0.9)
9-5	(6,11,16,21;1,1),(7,12,15,20;0.9,0.9)	(8,10,13,16;1,1),(9,11,12,15;0.9,0.9)	(7,10,13,16;1,1),(8,11,12,15,0.9,0.9)
6-9	(13,17,21,25;1,1),(14,18,20,24;0.9,0.9)	(15,18,21,24;1,1),(16,19,20,23;0.9,0.9)	(16,18,21,23;1,1),(17,19,20,22;0.9,0.9)
8-9	(14,17,20,23;1,1),(15,18,19,22;0.9,0.9)	(15,17,20,22;1,1),(16,18,19,21;0.9,0.9)	(16,18,21,23;1,1),(17,19,20,22;0.9,0.9)

Table 3

IT2F cost of activities (100 $ US)

ACT.	Experts		
	D_1	D_2	D_3
0-1	(12,17,22,27;1,1), (13,18,20,23;0.9,0.9)	(14,19,24,29;1,1), (15,20,23,28;0.9,0.9)	(14,19,24,29;1,1), (15,20,23,28;0.9,0.9)
0-2	(4,9,14,19;1,1), (5,10,13,18;.9,.9)	(4.5,9.5,14.5,19.5;1,1), (5.5,10.5,13.518.5;0.9,0.9)	(4,9,14,19,1), (5,10,13,18;0.9,0.9)
0-3	(2.5,7.5,12.5,17.5;1,1), (3.5,8.5,10.5,16.5,0.9,0.9)	(2.5,7.5,12.5,17.5;1,1), (3.5,8.5,11.5,16.5;0.9,0.9)	(3,7,11,15;1,1), (4,8,10,14,0.9,0.9)
1-4	(2,7,15,20;1,1), (3,7,12,18;0.9,0.9)	(2,7,12,17;1,1), (3,8,11,16;0.9,0.9)	(3,8,13,18;1,1), (4,9,12,17;0.9,0.9)
2-4	(13,20,27,34;1,1), (14,22,26,32;0.9,0.9)	(14,20,26,32;1,1), (15,21,25,31;0.9,0.9)	(13,19,25,31;1,1), (14,20,24,30;0.9,0.9)
3-5	(60,65,70,75;1,1), (62,66,68,73,0.9,0.9)	(55,60,65,70;1,1), (56,61,64,69;0.9,0.9)	(50,60,70,80;1,1), (51,61,69,79;0.9,0.9)
7-8	(16,20,24,28;1,1), (17,21,23,26;0.9,0.9)	(14,20,26,32;1,1), (15,21,25,31;0.9,0.9)	(12,20,28,36;1,1), (13,21,27,35;0.9,0.9)
2-5	(17,22,27,32;1,1), (18,23,25,30;0.9,0.9)	(15,20,25,30;1,1), (16,21,24,29;0.9,0.9)	(14,20,26,32;1,1), (15,21,25,31,0.9,0.9)
4-6	(7.5,12.5,17.5,22.5;1,1), (8.5,13.5,15.5,20;0.9,0.9)	(7.5,12.5,17.5,22.5;1,1), (8.5,13.5,16.5,21.5;0.9,0.9)	(7,10,13,16;1,1), (8,11,12,15;0.9,0.9)
3-7	(9,15,21,27;1,1), (10,17,20,25;0.9,0.9)	(10,15,20,25;1,1), (12,16,19,24,0.9,0.9)	(11,14,17,20;1,1), (12,15,16,19;0.9,0.9)
5-9	(14,20,26,32;1,1), (15,21,25,31;0.9,0.9)	(15.5,20.5,25.5,30.5;1,1), (16.5,21.5,24.5,29.5;0.9,0.9)	(15,20,25,30;1,1), (16,21,24,29,0.9,0.9)
6-9	(35,40,45,50;1,1), (36,41,44,49;0.9,0.9)	(35,40,45,50;1,1), (36,42,44,49;0.9,0.9)	(30,40,50,60;1,1), (31,41,49,59;0.9,0.9)
8-9	(24,30,36,42;1,1), (25,32,35,40;0.9,0.9)	(24,30,36,42;1,1), (26,32,35,40;0.9,0.9)	(24,30,36,42;1,1), (25,31,35,41;.9,.9)

Table 4

Linguistic variable for ratings of activities based on the risk, quality, and safety factors

Risk			Safety			Quality			ACT.
Dm_1	Dm_2	Dm_3	Dm_1	Dm_2	Dm_3	Dm_1	Dm_2	Dm_3	
L	ML	L	M	ML	M	ML	ML	L	1-0
ML	ML	M	ML	M	M	ML	M	M	2-0
MH	M	H	L	L	M	MH	M	MH	3-0
M	M	ML	L	MH	MH	ML	M	ML	4-1
MH	M	MH	M	MH	M	MH	H	MH	4-2
M	ML	M	VH	M	ML	M	MH	M	5-3
ML	ML	ML	MH	M	MH	ML	MH	MH	8-7
M	ML	M	ML	M	MH	M	ML	M	5-2
M	MH	MH	ML	M	ML	M	ML	MH	6-4
M	M	MH	ML	MH	M	M	MH	MH	7-3
MH	MH	H	M	MH	M	MH	H	H	9-5
H	MH	H	H	ML	M	H	MH	H	9-6
MH	MH	MH	H	M	M	H	MH	MH	9-8

Step 2. Normalizing decision matrixes by using Eqs. (12) and (13).

Step 3. Calculating IT2F entropy method based on the ratio of distances between the (N, N_{near}) and (N, N_{far}) by using these substeps:

Step 3-1. Computing the value of T_j^k employing Eqs. (14) and (15).

Step 3-2. Calculating the entropy value by using Eq. (16).

Step 3-3. Computing the degree of divergence utilizing Eq. (17).

Step 3-4. Obtaining weights of TCRSQ by using Eq. (18) for each decision matrix is shown in Table 5.

Table 5
Weight of important factors

Weight	$\tilde{\tilde{D}}_1$	$\tilde{\tilde{D}}_2$	$\tilde{\tilde{D}}_3$
Time	0.177	0.28	0.175
Cost	0.198	0.146	0.204
Risk	0.195	0.208	0.186
Quality	0.214	0.163	0.178
safety	0.214	0.203	0.257

Step 4. Computing normalized decision matrix multiplied by weight via of Eq. (19).

Step 5. Calculating each DM's weight through the extension of RPR under IT2F-environment.

Step 5-1. Constructing the IT2F average of group decision matrixes (S^*) and negative ideal solution (S) matrixes under IT2F through Eqs. (20-22).

Step 5-2. Calculating $\mu_{p^*}(\tilde{\tilde{s}}_{ij}^*, \tilde{\tilde{v}}_{ij}^k)$ and $\mu_{p^*}(\tilde{\tilde{v}}_{ij}^k, \tilde{\tilde{s}}_{ij}^-)$ using Eqs. (23-32).

Step 5-3. Determining the relative closeness for DMs by using Eq. (33) that is demonstrated in Table 6.

Table 6
Relative closeness

$\sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{P^*}^{p}(\tilde{\tilde{s}}_{ij}^*, \tilde{\tilde{v}}_{ij}^p)$	$\sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{P^*}^{p}(\tilde{\tilde{v}}_{ij}^p, \tilde{\tilde{s}}_{ij}^-)$	Ω_p
12.26	13.63	0.526
13.22	12.66	0.489
12.37	13.47	0.521

Step 5-4. Obtaining the final weight (FW) for DMs employing Eq. (34), the results are shown in Table 7.

Step 6. Aggregating DM's views via Eq. (35).

Step 7. Determining the positive ideal and negative ideal solutions via Eq. (36) and (37).

Table 7
DM's final weights

Decision-makers	Final weights
DM_1	0.3425
DM_2	0.3182
DM_3	0.3392

Step 8. Computing distance from positive and negative ideal solutions through Eqs. (38) and (39) as indicated in Table 8.

Step 9. Calculating the final ranking (*F*) using Eq. (40), which is depicted in Table 8.

Table 8
Final ranking

Path	D^+	D^-	Final results	Final ranking
0-1-4-6-9	0.4702	0.6636	1.5719	2
0-2-5-9	0.7339	0.3892	1.5462	5
0-2-4-6-9	0.4319	0.7126	1.5887	1
0-3-5-9	0.4747	0.6509	1.5603	4
0-3-7-8-9	0.5694	0.5625	1.5653	3

Sensitivity analysis: In this part, the evaluation is done based on efficient factors; that is why the critical path is calculated utilizing time factor; then it is computed based on time and cost factors and so on. The results of sensitivity analysis have been presented in Tables 9-1 and 9-2. Also, the graphical results have been illustrated in Figure 4. As given in Tables 9-1 and 9-2 and Figure 4, all experiments have been changed. The critical path depends on each factor; in other words, each path's criticality is altered in all experiments by changing efficient factors. When time factor is considered to determine the critical path except for the critical path (path 3), all of other path is altered as presented in Table 9.

Other factors that have been considered in this paper are essential as much as time or more important time.

Table 9
Sensitivity analysis

Paths	Time	Time, cost	Time, risk	Time, quality	Time, safety	Time, cost, risk	Time, cost, quality	Time, cost, safety
0-1-4-6-9	4	3	3	2	4	3	3	3
0-2-5-9	3	5	5	4	3	5	4	5
0-2-4-6-9	1	1	1	1	1	1	1	1
0-3-5-9	5	4	4	5	5	4	5	4
0-3-7-8-9	2	2	2	3	2	2	2	2
Paths	**Time, risk, quality**	**Time, risk, safety**	**Time, quality, safety**	**Time, cost, risk, quality**	**Time, cost, risk, safety**	**Time, cost, quality, safety**	**Time, risk, quality, safety**	
0-1-4-6-9	5	3	2	2	3	4	3	
0-2-5-9	2	4	1	5	5	3	4	
0-2-4-6-9	4	1	3	1	1	1	1	
0-3-5-9	1	5	4	4	4	5	5	
0-3-7-8-9	3	2	5	3	2	2	2	

Fig. 4. Sensitivity analysis

5. Conclusion

In this paper, a new analysis model under an IT2F-environment has been developed in order to better deal with the uncertainty of practical mega projects. As a matter of fact, IT2FSs are more beneficial than type-1 fuzzy sets. This is because the membership grade of type-1 fuzzy sets belongs to [0,1], whereas the membership grade of T2FS is a type-1 fuzzy set. Time and cost factors have been presented as quantitative criteria, whereas risk, quality, and safety are introduced as qualitative criteria expressed by linguistic variables and their IT2F equivalents. A new method has also been developed to determine each DM's weight by an extension of RPR under an IT2F-environment and used for aggregating DMs' judgments. Moreover, a new IT2F-entropy method for assessing the importance of efficient factors such as TCRSQ has been introduced using the concept of ratio $F,F_{far,}$ and F,F_{near}. A practical example adopted from the literature has been resolved to demonstrate better the proposed model's implementation process and its applicability. This proposed analysis model has provided experts and planners with a useful way to properly determine the critical path in real-world mega projects. For future study, the introduced method can be applied to other MCDM fields. Besides, other weighting methods, such as eigenvector and cross-entropy, can be used in the model.

References

Amiri, M., & Golozari, F. (2011). Application of fuzzy multi-attribute decision making in determining the critical path by using time, cost, risk, and quality criteria. International Journal of Advanced Manufacturing Technology, 54(1), 393-401.

Amiri, M., Zandieh, M., Vahdani, B., Soltani, R. & Roshanaei, V., (2010). An integrated eigenvector–DEA–TOPSIS methodology for portfolio risk evaluation in the FOREX spot market. Expert systems with applications, 37(1), 509-516.

Castro, J. R., Castillo, O., Melin, P., & Rodríguez-Díaz, A. (2009). A hybrid learning algorithm for a class of interval type-2 fuzzy neural networks. Information Sciences, 179(13), 2175-2193.

Chanas, S., & Zieliński, P. (2001). Critical path analysis in the network with fuzzy activity times. Fuzzy sets and systems, 122(2), 195-204.

Chen, S. M., & Lee, L. W. (2010). Fuzzy multiple attributes group decision-making based on the ranking values and the arithmetic operations of interval type-2 fuzzy sets. Expert Systems with applications, 37(1), 824-833.

Chen, S. P. (2007). Analysis of critical paths in a project network with fuzzy activity times. European Journal of Operational Research, 183(1), 442-459.

Chen, S. P., & Hsueh, Y. J. (2008). A simple approach to fuzzy critical path analysis in project networks. Applied Mathematical Modelling, 32(7), 1289-1297.

Dorfeshan, Y., & Mousavi, S.M. (2020). A novel interval type-2 fuzzy decision model based on two new versions of relative preference relation-based MABAC and WASPAS methods (with an application in aircraft maintenance planning). Neural Computing and Applications, 32, 3367–3385.

Dorfeshan, Y., Mousavi, S.M., Vahdani, B., & Siadat, A. (2019). Determining project characteristics and critical path by a new approach based on modified NWRT method and risk assessment under an interval type-2 fuzzy environment, 26(4), 2579-2600.

Dorfeshan, Y., Mousavi, S.M., Zavadskas, E.K., & Antucheviciene, J. (2021). A new enhanced ARAS method for critical path selection of engineering projects with interval type-2 fuzzy sets. International Journal of Information Technology & Decision Making, 20(1), 37–65.

Eshghi, A., Mousavi, S.M., & Mohagheghi, V., (2019). A new interval type-2 fuzzy approach for analyzing and monitoring the performance of megaprojects based on earned value analysis (with a case study), Neural Computing and Applications, 31, 5109–5133.

Foroozesh, N., Tavakkoli-Moghaddam, R., Mousavi, S. M., & Vahdani, B. (2019). A new comprehensive possibilistic group decision approach for resilient supplier selection with mean–variance–skewness–kurtosis and asymmetric information under interval-valued fuzzy uncertainty. Neural Computing and Applications, 31(11), 6959-6979.

Foroozesh, N., Tavakkoli-Moghaddam, R., & Mousavi, S. M. (2017). Resilient supplier selection in a supply chain by a new interval-valued fuzzy group decision model based on possibilistic statistical concepts. Journal of Industrial and Systems Engineering, 10(2), 113-133.

Foroozesh, N., Tavakkoli-Moghaddam, R., & Mousavi, S. M. (2018). A novel group decision model based on mean–variance–skewness concepts and interval-valued fuzzy sets for a selection problem of the sustainable warehouse location under uncertainty. Neural Computing and Applications, 30(11), 3277-3293.

Gitinavard, H., Mousavi, S.M. & Vahdani, B., (2017a). Soft computing-based new interval-valued hesitant fuzzy multi-criteria group assessment method with last aggregation to industrial decision problems. Soft Computing, 21(12), 3247-3265.

Gitinavard, H., Mousavi, S.M. & Vahdani, B., (2017b). Soft computing based on hierarchical evaluation approach and criteria interdependencies for energy decision-making problems: A case study. Energy, 118, 556-577.

Haghighi, M.H., Mousavi, S.M., & Mohagheghi, V., (2019). A new soft computing model based on linear assignment and linear programming technique for multidimensional analysis of preference with interval type-2 fuzzy sets, Applied Soft Computing, 77, 780–796.

Hillier F.S., G.J. Liebermann. *(2001). Introduction to Operations Research, seventh edition, McGraw Hill, Singapore.*

Jammeh, E. A., Fleury, M., Wagner, C., Hagras, H., & Ghanbari, M. (2009). Interval type-2 fuzzy logic congestion control for video streaming across IP networks. IEEE Transactions on Fuzzy Systems, 17(5), 1123-1142.

Kaur, P., & Kumar, A. (2014). Linear programming approach for solving fuzzy critical path problems with fuzzy parameters. Applied Soft Computing, 21, 309-319.

Kiracı, K., & Akan, E. (2020). Aircraft selection by applying AHP and TOPSIS in interval type-2 fuzzy sets. Journal of Air Transport Management, 89, 101-114.

Krajewski L.J. & L.P. Ritzman (2005). Operations Management: Process and Value chains, seventh edition, Prentice-Hill, New Jersey, 2005.

Kurihara, K., & Nishiuchi, N. (2002). Efficient Monte Carlo simulation method of GERT-type network for project management. Computers & Industrial Engineering, 42(2), 521-531.

Liang, Q., & Mendel, J. M. (2000). Interval type-2 fuzzy logic systems: theory and design. IEEE Transactions on Fuzzy systems, 8(5), 535-550.

Liang, S. K., Yang, K. L., & Chu, P. (2004). Analysis of fuzzy multiobjective programming to CPM in project management. Journal of Statistics and Management Systems, 7(3), 597-609.

Liao, T. W. (2015). Two interval type 2 fuzzy TOPSIS material selection methods. Materials & Design, 88, 1088-1099.

Liu, P., & Gao, H. (2020). A novel green supplier selection method based on the interval type-2 fuzzy prioritized choquet bonferroni means. IEEE/CAA Journal of Automatica Sinica, Article in press, Doi: 10.1109/JAS.2020.1003444.

Madhuri, K. U., Siresha, S., & Shankar, N. R. (2012). A new approach for solving fuzzy critical path problem using LL fuzzy numbers. Applied Mathematical Sciences, 6(27), 1303-1324.

Mehlawat, M. K., & Gupta, P. (2016). A new fuzzy group multi-criteria decision making method with an application to the critical path selection. The International Journal of Advanced Manufacturing Technology, 83(5-8), 1281-1296.

Mendel, J. M., & Wu, H. (2006). Type-2 fuzzistics for symmetric interval type-2 fuzzy sets: Part 1, forward problems. IEEE Transactions on Fuzzy Systems, 14(6), 781-792.

Mendel, J. M., John, R. I., & Liu, F. (2006). Interval type-2 fuzzy logic systems made simple. IEEE transactions on fuzzy systems, 14(6), 808-821.

Mohagheghi, V., & Mousavi, S.M., (2019). An analysis approach to handle uncertain multi-criteria group decision problems in the framework of interval type-2 fuzzy sets theory, Neural Computing and Applications, 31(8), 3543–3557.

Mohagheghi, V., Mousavi, S.M. and Vahdani, B., (2015). A new optimization model for project portfolio selection under interval-valued fuzzy environment. Arabian Journal for Science & Engineering, 40(11), 3351-3361.

Mohagheghi, V., Mousavi, S.M. & Vahdani, B., (2017). Analyzing project cash flow by a new interval type-2 fuzzy model with an application to construction industry. Neural Computing and Applications, 28(11), 3393-3411.

Mon, D. L., Cheng, C. H., & Lu, H. C. (1995). Application of fuzzy distributions on project management. Fuzzy sets and systems, 73(2), 227-234.

Moradi, N., Mousavi, S.M., & Vahdani, B., (2018). An interval type-2 fuzzy model for project-earned value analysis under uncertainty, Journal of Multiple-Valued Logic and Soft Computing, 30, 79–103.

Mousavi, S. M., Antuchevičienė, J., Zavadskas, E. K., Vahdani, B., & Hashemi, H. (2019). A new decision model for cross-docking center location in logistics networks under interval-valued intuitionistic fuzzy uncertainty. Transport, 34(1), 30-40.

Mousavi, S.M. & Vahdani, B., (2016). Cross-docking location selection in distribution systems: a new intuitionistic fuzzy hierarchical decision model. International Journal of computational intelligence Systems, 9(1), 91-109.

Mousavi, S.M., Vahdani, B., Tavakkoli-Moghaddam, R. & Tajik, N., (2014). Soft computing based on a fuzzy grey group compromise solution approach with an application to the selection problem of material handling equipment. International Journal of Computer Integrated Manufacturing, 27(6), 547-569.

Mousavi, S.M., Vahdani, B., Tavakkoli-Moghaddam, R., Ebrahimnejad, S. & Amiri, M., (2013). A multi-stage decision-making process for multiple attributes analysis under an interval-valued fuzzy environment. The International Journal of Advanced Manufacturing Technology, 64(9-12), 1263-1273.

Nasir, D., McCabe, B., & Hartono, L. (2003). Evaluating risk in construction–schedule model (ERIC–S): construction schedule risk model. Journal of construction engineering and management, 129(5), 518-527.

Ock, J. H., & Han, S. H. (2010). Measuring risk-associated activity's duration: A fuzzy set theory application. KSCE Journal of Civil Engineering, 14(5), 663-671.

San Cristobal, J. R. (2012). Critical path definition using multicriteria decision making: PROMETHEE method. Journal of Management in Engineering, 29(2), 158-163.

Sari, I. U., & Kahraman, C. (2015). Interval type-2 fuzzy capital budgeting. International Journal of Fuzzy Systems, 17(4), 635-646.

Shukla, A. K., & Muhuri, P. K. (2019). General type-2 fuzzy decision making and its application to travel time selection. Journal of Intelligent & Fuzzy Systems, 36(6), 5227-5244.

Vahdani, B. & Zandieh, M., (2010). Selecting suppliers using a new fuzzy multiple criteria decision model: the fuzzy balancing and ranking method. International Journal of Production Research, 48(18), 5307-5326.

Vahdani, B., Mousavi, S.M. & Ebrahimnejad, S., (2014). Soft computing-based preference selection index method for human resource management. Journal of Intelligent & Fuzzy Systems, 26(1), 393-403.

Vahdani, B., Razmi, J. & Tavakkoli-Moghaddam, R., (2012). Fuzzy possibilistic modeling for closed loop recycling collection networks. Environmental Modeling & Assessment, 17(6), pp.623-637.

Vahdani, B., Zandieh, M. & Roshanaei, V., (2011). A hybrid multi-stage predictive model for supply chain network collapse recovery analysis: a practical framework for effective supply chain network continuity management. International Journal of Production Research, 49(7), 2035-2060.

Yılmaz, H., & Kabak, Ö. (2020). Prioritizing distribution centers in humanitarian logistics using type-2 fuzzy MCDM approach. Journal of Enterprise Information Management, 33(5), 1199-1232.

Zadeh, L. A. (1965). Fuzzy sets. Information and control, 8(3), 338-353.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. Information sciences, 8(3), 199-249.

Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. Fuzzy sets and systems, 1(1), 3-28.

Zammori, F. A., Braglia, M., & Frosolini, M. (2009). A fuzzy multi-criteria approach for critical path definition. International Journal of Project Management, 27(3), 278-291.

Zamri, N., & Abdullah, L. (2013). A new linguistic variable in interval type-2 fuzzy entropy weight of a decision making method. Procedia Computer Science, 24, 42-53.

Zareei, A., Zaerpour, F., Bagherpour, M., Noora, A. A., & Vencheh, A. H. (2011). A new approach for solving fuzzy critical path problem using analysis of events. Expert Systems with Applications, 38(1), 87-93.

Zhang, Z., & Zhang, S. (2013). A novel approach to multi attribute group decision making based on trapezoidal interval type-2 fuzzy soft sets. Applied Mathematical Modelling, 37(7), 4948-4971.

This article can be cited:
Dorfeshan, Y., Mousavi, S., Vahdani, B. (2022). A New Analysis of Critical Paths in Mega Projects with Interval Type-2 Fuzzy Activities by Considering Time, Cost, Risk, Quality, and Safety Factors. Journal of Optimization in Industrial Engineering, 15(1), 145-160.

http://www.qjie.ir/article_686012.html
DOI: 10.22094/joie.2021.761.1485