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Sensitivity Analysis in the QUALIFLEX and VIKOR Methods

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Abstract

The sensitivity analysis for multi-attribute decision making (MADM) problems is important for two reasons: First, the decision matrix as the source of the results of a decision problem is inaccurate because it sorts the alternatives in each criterion inaccurately. Second, the decision maker may change his opinions in a time period because of changes in the importance of the criteria and in the policy of the organization over time. This in turn makes problem solving really time-consuming. Therefore, the best solution is to do sensitivity analysis. In this regard, this paper considers a sensitivity analysis in the QUALIFLEX method which is a compromise ranking method used for multi-criteria decision making (MCDM).

Keywords: Sensitivity Analysis; QUALIFLEX, VIKOR; Multi-criteria Decision Making; Multi-attribute Decision Making.

1. Introduction

Generally, as organizations have limited resources for fulfilling their goals, managers should always make important decisions regarding selecting the best option among different alternatives. Simple examples include deciding about what to buy, how to arrive to a place, where to go, and whom to employ. Decisions are made about various issues from logistics management, customer relationship management, marketing to production planning.

The process of decision making involves complications. Specifically, decision makers sometimes should deal with multi-attribute decision making (MADM) problems. MADM refers to making preference decision over the available alternatives that are characterized by multiple, usually conflicting, attributes. Inter and intra- attribute comparisons between alternatives enable decision makers to make the final decision (Lu, 2007).

Mathematically, a typical MADM method can be formulated as follows:

$$\begin{cases} Max(Min) A_1, A_2, \dots, A_m \\ S.t: \quad C_1, C_2, \dots, C_n \end{cases} \tag{1}$$

Where $A = (A_1, A_2, \dots, A_m)$ denotes m alternatives and C= $(C_1, C_2, \dots, C_n)$ represents n attributes (often called criteria) for characterizing a decision situation. The *select* here is normally based on maximizing a multi-attribute value (or utility) function elicited from the stakeholders.

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The basic information involved in this model can be expressed by this matrix:

$$D = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ A_1 & x_{11} & x_{12} & \cdots & x_{1n} \\ A_2 & x_{12} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & x_{m1} & x_{m2} & \cdots & x_{mn} \end{matrix}$$

$$W = [w_1 w_2 \dots w_n]$$

where $A_1, A_2, \dots, A_m$ are alternatives from which decision makers choose; $C_1, C_2, \dots, C_n$ are attributes with which alternative performances are measured and x_{ij} i=1,…,m, j=1,…,n, is the rating of alternative A_i with respect to attribute C_j; and W_j is the weight of attribute C_j.

Alternatives, Criteria and weights are the three important factors in a MADM method. As weights of criteria are very important, any change in the weighting made by the Decision maker can change the ranking of alternatives. There are several methods for solving MADM problems. The difference between them lies in the approach to omitting alternatives.

The outline of the present paper is as follows: an introduction to the QUALIFLEX and VIKOR methods is presented in section 2, the sensitivity analysis of the QUALIFLEX and VIKOR methods are discussed in section3, numerical examples are provided in section4 and finally section 5 presents conclusions.

3. Sensitivity Analysis

The literature on sensitivity analysis began with Evans (Evans. 1984). He focused on the sensitivity of an optimal decision to changes in the probabilities of the states of nature and the development of "confidence spheres" to bound arbitrary parametric changes in the probability vector. Then Schneller and Sphicas (1985)corrected the closed-form distance formulae and showed it was easily derived from Evans' data. Moreover, Isaacs (1963) explored the general question of sensitivity to subjective probability estimates and derived an analytical representation of sensitivity. In some other studies, problems of applying sensitivity analyses were explored and methods were also suggested for resolving conflict of choice under conditions of high sensitivity. Barron and Schmidt (1998) presented two simple computational procedures for sensitivity analysis of additive multi-attribute value models that yield variations in attribute weights (scaling constants). They developed the two methods of entropy-based and least squares procedure. Rios and French (1991) also introduced a framework for sensitivity analysis in multi-objective decision making within a Bayesian context. Rios and Salhi (2003) investigated an opportunistic approach aimed at reducing the number of optimization problems solved in the original framework and an alternative framework based on distance analysis. Triantaphyllou and Sanchez (1997) developed a methodology for performing sensitivity analysis on the weights of the decision criteria and the performance values of the alternatives expressed in terms of the decision criteria.

3.1. Sensitivity Analysis of the Present Study

In this paper, we use weight vector $W = [w_1 w_2 \dots w_n]$and change weight of the $k$thcriteriaw'_k. Then we find the impact of this change on the other weights$W' = (w'_1, w'_2, \dots, w'_n)$ and the final score P' of the VIKOR alternatives.

We assume that the total weights are normalized and the total amount of the weights is equal to1($\sum_{j=1}^{n} w_j = 1$).

Theorem1. In an MADM model if the weight of the kth criterion changes with the amount ofΔ_k, the weight of other criteria will change with the amount of$\Delta_j$as follows:

$$\Delta_j = \frac{\Delta_k w_j}{w_k - 1} \qquad j = 1,2,\dots,n\ , j \neq k \tag{11}$$

Proof. The new weights of the criterion k $w'_k$and the weights of other criteriaw'_j are given as:

$$w'_k = w_k + \Delta_k \tag{12}$$

$$w'_j = w_j + \Delta_j \qquad j = 1,2,\dots,n\ , j \neq k \tag{13}$$

When$\sum_{j=1}^{n} w_j = 1$, we will have:

$$\sum_{j=1}^{n} w'_j = \sum_{j=1}^{n} w_j + \sum_{j=1}^{n} \Delta_j \quad \Rightarrow \sum_{j=1}^{n} \Delta_j = 0$$

Then:

$$\Delta_k = -\sum_{\substack{j=1\\ j\neq k}}^{n} \Delta_j$$

If we assume$\Delta_j = \frac{\Delta_k w_j}{w_k - 1}$, the following formulae is correct:

$$\Delta_k = -\sum_{\substack{j=1\\ j\neq k}}^{n} \Delta_j \Rightarrow -\Delta_k = \sum_{\substack{j=1\\ j\neq k}}^{n} \Delta_j = \sum_{\substack{j=1\\ j\neq k}}^{n} \frac{\Delta_k w_j}{w_k - 1} = \frac{\Delta_k}{w_k - 1}\sum_{\substack{j=1\\ j\neq k}}^{n} w_j = \frac{\Delta_k}{w_k - 1}(1 - w_k) = -\Delta_k$$

Conclusion. If the weight of criterion k changes tow'_k, the weights of other criteriaw'_jchange as follows:

$$w'_j = w_j + \Delta_j = w_j + \frac{\Delta_k w_j}{w_k - 1} = \frac{w_j(w_k - 1) + \Delta_k w_j}{w_k - 1}$$

$$\Rightarrow w'_j = \frac{w_j(w_k - 1) + \Delta_k w_j}{w_k - 1} \tag{14}$$

Finally, the values of $w'_j$for two conditions of$j \neq k$and $j = k$will be as follows:

$$w'_j = \begin{cases} w_j + \Delta_k & if\ j = k \\ \frac{w_j(w_k - 1) + \Delta_k w_j}{w_k - 1} & if\ j \neq k \end{cases} \tag{15}$$

3.2. The Use of Sensitivity Analysis in the VIKOR Method

We introduce the VIKOR distance as follows:

$$d_{ij} = \frac{f_j^* - f_{ij}}{f_j^* - f_j^-}$$

We want to find the amount of changes in the final scores of the attributes as well as the final scores of the attributes after changingΔ_k of the weight of criteria k.

Theorem2. In a VIKOR multi-attribute decision making model, if the weight of the criteria k changes with the amount ofΔ_k, the utility measure will be as follows:

$$S'_i = \sum_{j=1}^{n} w'_j \times d_{ij} = \sum_{\substack{j=1\\ j\neq k}}^{n} \frac{(1 - w_k - \Delta_k).w_j}{1 - w_k} \times d_{ij} + (w_k + \Delta_k) \times d_{ij}$$
$$= \left(\frac{(1 - w_k - \Delta_k).w_j}{1 - w_k}\right) \times \sum_{\substack{j=1\\ j\neq k}}^{n} w_j \times d_{ij} + w_k \times d_{ij} + \Delta_k \times d_{ij}$$
$$= \left(\frac{(1 - w_k - \Delta_k).w_j}{1 - w_k}\right) \times \sum_{\substack{j=1\\ j\neq k}}^{n} w_j \times d_{ij} + \frac{\Delta_k}{1 - w_k} \times d_{ij} + \Delta_k \times d_{ij}$$

$$\Rightarrow S'_i = \left(1 - \frac{\Delta_k}{1 - w_k}\right) \times S_i + \frac{\Delta_k}{1 - w_k} \times d_{ij} \tag{16}$$

$R_i$will be exactly the same asS_i:

$$R'_i = \left(1 - \frac{\Delta_k}{1 - w_k}\right) \times R_i + \frac{\Delta_k}{1 - w_k} \times d_{ij} \tag{17}$$

$per_2 = {}^{1}/_{2}$
$per_3 = {}^{-3}/_{2}$
$per_4 = -2$

$per_5 = {}^{-1}/_{2}$
$per_6 = {}^{9}/_{4}$

4.2. The VIKOR Numerical Example

A mountain climber (beginner) must choose an alternative from a set of three alternatives, i.e. destinations$\{A_1, A_2, A_3\}$. The evaluation of the alternatives is presented in Table 4. Let's suppose that both evaluation criteria—risk and altitude— are equally important, i.e. the weight of criteria are$w_i = {}^{1}/_{2}$(Martel et al., 2005).

Table 4
Problem f

Criteria	Alternatives		
	A_1	A_2	A_3
f_1 Risk, subjective evaluation, scale: 1,2,3,4,5	1	2	5
f_2 Altitude, evaluated in meters above the sea	3000	3750	4500

$f_1^* = 1\,, f_1^- = 5$
$f_2^* = 4500\,, f_2^- = 3000$

After solving the problem, the values of S_i, R_i and Q_i will be as follows in Table5.

Table5
Values of*Si, Ri* and *Qi*

	Alternatives			Ranking
	A_1	A_2	A_3	
S	0.5	0.375	0.5	$A_2, A_1 \approx A_3$
R	0.5	0.25	0.5	$A_2, A_1 \approx A_3$
Q	1	0	1	$A_2, A_1 \approx A_3$

Sensitivity Analysis of the VIKOR Numerical Example

We assume that the weight of criterion 1 changes to 0.2 ($w_1' = 0.2$).

$$w_2' = \frac{0.5 \times (1 - 0.5 + 0.3)}{1 - 0.5} = 0.8$$

Then$S_1', S_2'$and S_3' will be as follows:

$$S_1' = 0.2 \times \frac{1-1}{5-1} + 0.8 \times \frac{3000-4500}{3000-4500} = 0.8$$

$$S_2' = 0.2 \times \frac{2-1}{5-1} + 0.8 \times \frac{3750-4500}{3000-4500} = 0.45$$

$$S_3' = 0.2 \times \frac{5-1}{5-1} + 0.8 \times \frac{4500-4500}{3000-4500} = 0.2$$

The regret measures ofR_1', R_2' and $R_3'$are as follows:

$R_1' = \max\{0\,, 0.8\} = 0.8$
$R_2' = \max\{0.05\,, 0.4\} = 0.4$
$R_3' = \max\{0.2\,, 0\} = 0.2$

Finally, the VIKOR indices will be as follows:

$$Q_1' = 0.5 \times \frac{0.8-0.8}{0.2-0.8} + 0.5 \times \frac{0.8-0.8}{0.2-0.8} = 0$$

$$Q_2' = 0.5 \times \frac{0.4-0.8}{0.2-0.8} + 0.5 \times \frac{0.45-0.8}{0.2-0.8} = 0.625$$

$$Q_3' = 0.5 \times \frac{0.2-0.8}{0.2-0.8} + 0.5 \times \frac{0.2-0.8}{0.2-0.8} = 0.5$$

$$Q_1' < Q_3' < Q_2' \;\Rightarrow\; A_1 > A_2 > A_3$$

5. Conclusions

Decision making can be categorized into two different categories: Multi-objective decision making problems and MADM problems. In multi-objective decision making problems, decision makers are looking for the solution which has the best results with different objectives, but in MADM problems decision makers choose the best alternative based on different attributes. All MADM problems have a decision matrix which shows alternatives and attributes. Sometimes the attributes are conflicting so that the maximization of one attribute will result in the minimization of other attributes.

In the classical techniques of MADM, it is often assumed that all used data (such as weights of attributes, efficiency of alternatives against attribute etc.) are deterministic, thus final scores or utility of alternatives are obtained by solving MADM. However, in reality data of decision making problems are changing. Therefore, after solving decision making problems, usually a sensitivity analysis must be done for them. In this paper, we developed sensitivity analysis for the QUALIFLEX and VOKOR methods and proposed a method based on changes in the weights.

6. References

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